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Studies of Empirical Formulas for Total Reaction Cross Sections at 14–15 MeV Neutrons

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Abstract. Around 14–15 MeV neutron incident energy elastic, nonelastic and total nuclear reaction cross sections have been investigated by many authors in literature. But theoretical calculations have not been adequate because of the character of the nuclear structure is not exactly known. Flerov and Talyzin obtained an empirical nonelastic cross section formula and also they found that the total cross section is quite different a function of atomic weight for 14-15 MeV neutrons. Lastly, Tel et al. developed a new empirical formula for (n,2n) and (n,d) cross sections by using Flerov and Talyzin expression for the inelastic reaction at 14–15 MeV neutrons. In the calculations, the scattering theory was used and the obtained new empirical formula was found as better agreement with experimental data.

In this study, the total reaction cross sections, around 14–15 MeV energy, for target nuclei in the mass region (A:1-238) have been investigated. The total cross sections calculations have been done by using optical model parameters and using SCAT2 code. The obtained theoretical results compared with the taken from EXFOR experimental nuclear data library. Consequently, calculations of the total cross sections in the present study may provide a reference to neutron reactions considering the lack of experimental data for around 14–15 MeV energy.

1 Introduction

One of the methods for determining nuclear radii is based on measurements of the total cross sections of nuclei for fast neutrons. Nuclear radii are most likely to be calculable from such measurements if the neutron wavelength divided by 2π is small compared to the nuclear radius, but not small enough that the nucleus is transparent for the neutrons used [1].

Feshbach and Weisskopf have proposed a schematic theory of nuclear cross sections which should be applicable for neutrons of the energy used in the present experiment. According to this theory the square root of the total cross section should be approximately a linear function of the nuclear radius [2, 3].

According to the semitransparent model of the nucleus the dependence of the cross section for inelastic interactions σ_{in} on the radius of the nucleus for neutrons with energies of several millions of electron volts should have maxima and minima which become less distinct at higher energies. It would be expected that for neutrons with energies of 14 MeV the cross section for inelastic reactions would be a monotonic function of the nuclear radius [2–5].

Flerov and Talyzin obtained an empirical nonelastic cross section formula and also they found that the total cross section is quite different a function of atomic weight for 14–15 MeV neutrons [6]. Lastly, Tel *et al.* developed a new empirical formula for (n,2n) and (n,d) cross sections by using Flerov and Talyzin expression for the inelastic reaction at 14–15 MeV neutrons [7, 8]. In the calculations, the scattering theory was used and the obtained new empirical formula was found as better agreement with experimental data.

2 Materials and Methods

A theory for total neutron cross sections was developed by Feshbach and Weisskopf [2–4] under the assumption that the incident neutron and the target nucleus immediately form a compound nucleus. The resulting total cross sections decrease monotonically with increasing energy.

$$\sigma_{\text{tot}} \simeq \begin{cases} \dfrac{4\pi}{kK} & \lambda \gg R \\ 2\pi(R+\lambda)^2 & \lambda \ll R \end{cases} . \tag{1}$$

Here $k = \lambda^{-1}$ is the wave number of the incident neutron; λ is the reduced wavelength, $K \simeq 10^{13}$ cm^{-1} is the wave number of the neutron in the interior of the nucleus; and R is the nuclear radius.

The regular maxima and minima exhibited by the experimental results seem to indicate an interference of the incident wave with an outgoing one, suggesting that the neutron wave is not completely absorbed into collective motion in one passage through the nucleus. Feshbach and Weisskopf [2, 3] calculated the total neutron cross section with the following model. The nucleus is replaced by a potential well with a complex potential

$$V_r = \begin{cases} -V_0(1+i\xi) & r < R \\ 0 & r > R \end{cases} , \tag{2}$$

where V_0 is a real number giving the depth of the well and ξ is a parameter which indicates an "absorption" for neutrons within nuclear matter. This absorption is introduced in order to describe the formation of the compound nucleus.

Flerov and Talyzin found that the total cross section σ_{tot} is quite different a function of atomic weight for 14–15 MeV neutrons [6]. The empirical σ_{ne} nonelastic cross section of Flerov and Talyzin [6] is expressed as follows (in barn):

$$\sigma_{\text{ne}} = \pi \left(0.12A^{1/3} + 0.21\right)^2 . \tag{3}$$

Flerov and Talyzin investigated that the inelastic neutron collision cross-section for some elements and they obtained empirical the relation as follow (in cm):

$$\sqrt{\frac{\sigma_{\text{in}}}{\pi}} = \left(1.2A^{1/3} + 2.1\right) 10^{-13} . \tag{4}$$

The cross section for inelastic interactions of neutrons with nuclei in almost all the elements measured is a monotonic function of the atomic weight. According to the quantum-mechanical scattering theory, the non-elastic cross section is expressed by

$$\sigma_{\text{ne}} = \pi \left(R + \lambda\right)^2 , \tag{5}$$

where λ is the reduced wavelength. If the radius R of target nucleus is entered into Eq. (4), the following formula can be written

$$\sqrt{\frac{\sigma_{\text{ne}}}{\pi}} = \left(1.2A^{1/3} + \lambda\right) . \tag{6}$$

The reduced wavelength is given as below

$$\lambda = \frac{hc}{\sqrt{2mc^2E}} , \tag{7}$$

where E and m stand for the energy and mass of the incident neutron, respectively. c is the speed of light, $\hbar$ is the reduced Planck's constant. In Eq. (6), the reduced wavelength for incident neutrons with 14 MeV energy is equal to 1.2 fm, and this value is equal to the value of reduced radius. Therefore, the non-elastic cross section given in Eq. (5) can be expressed as

$$\sigma_{\text{ne}} = \pi r_0^2 \left(A^{1/3} + 1\right)^2 . \tag{8}$$

Using scattering theory, Tel *et al.* developed a new empirical formula for (n,2n) and (n,d) cross sections for the inelastic reaction at 14–15 MeV neutrons [7, 8]. In the calculations, Flerov and Talyzin expression was used and the obtained new empirical formulas were found as better agreement with experimental data. The obtained Tel *et al.* new empirical formula as follow for (n,2n) reaction cross sections at 14–15 MeV neutrons (in mb),

$$\sigma_{\text{n,2n}} = 37112 \left(0.12A^{1/3} + 0.21\right)^2 s1.7 \tag{9}$$

and for (n,d) reaction cross sections at 14–15 MeV neutrons (in barn)

$$\sigma_{\text{n,d}} = \left(0.12A^{1/3} + 0.21\right)^2 \left(0.016 - 0.239s + 1.199s^2\right) , \qquad (10)$$

where s is asymmetry parameter ($s = (N - Z)/A$), A is mass number, N is neutron number and Z is proton number of target nuclei.

3 Results and Discussion

In this study, the total reaction cross sections, around 14–15 MeV neutron energy, for target nuclei in the mass regions $1 \leq A \leq 238$ have been investigated. We analyzed total reaction cross sections depending with mass number

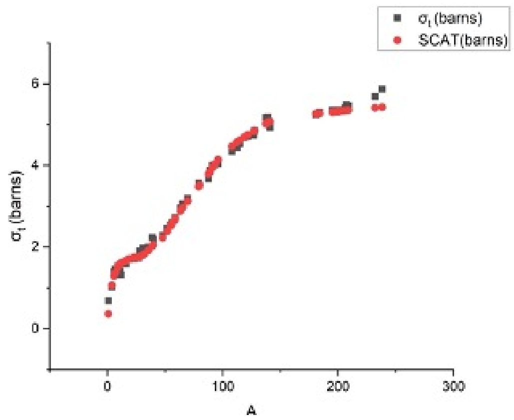

Figure 1. Total cross section versus A.

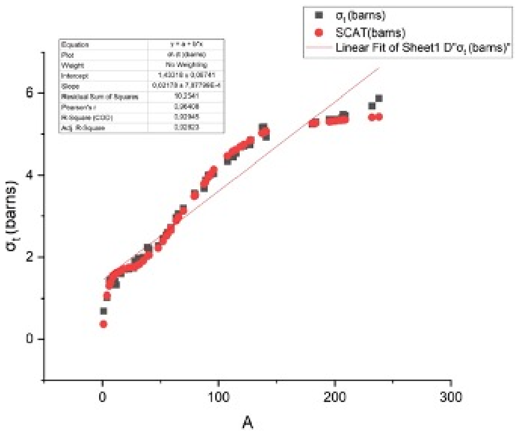

Figure 2. Total cross section versus A.

A. Theoretical calculations have been done using SCAT2 code and by using optical model parameters.The obtained theoretical results compared with the taken from EXFOR experimental nuclear data library. And also, we investigated linear, exponential and polynomial fitting methods and which fitting procedure was the most suitable. The correlation coefficients R^2 were determined, and the obtained results have been presented in Figures 1–4.

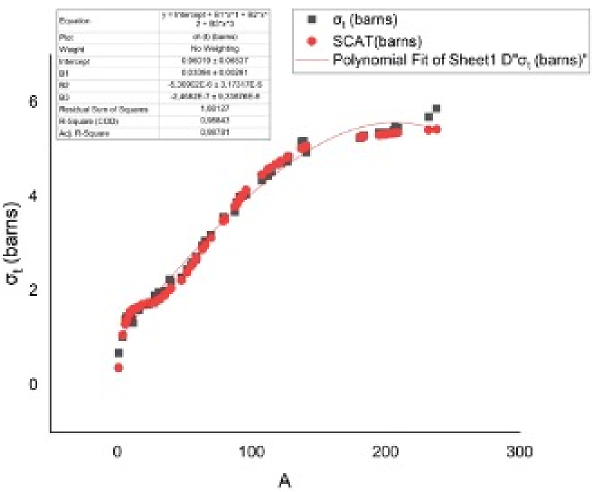

Figure 3. Total cross section versus A.

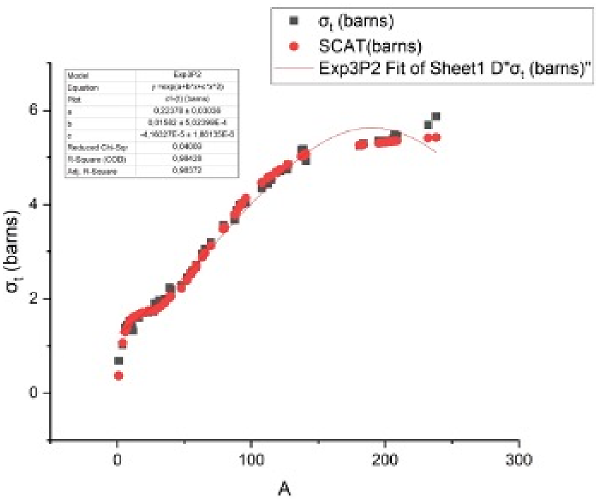

Figure 4. Total cross section versus A.

4 Conclusion

In the present study, we concluded that theoretical calculations for the total reaction cross sections by using optical model parameters with SCAT2 computer code have been found good agreement with experimental values the taken from EXFOR data library. All fitting procedures' results and R^2 values are given in Table 1. We obtained that the exponential and polynomial fitting methods give well-fitting results for total cross sections at 14–15 MeV neutrons. The resulting correlation coefficient R^2 is the lowest for linear fittings method. We concluded that total reaction cross sections depending with atomic weight increasing (mass number A) is the lowest for linear fittings for around 14–15 MeV neutron energy. The obtained results for the total cross sections may provide a reference to neutron reactions considering the lack of experimental data for around 14-15 MeV neutron energy.

Table 1. Fitting results for Total cross sections at 14–15 MeV neutrons

Fit	Equation	R^2
Linear fit	$y = 1.433x + 0.022$	0.92945
Polynomial fit	$y = 0.960 + 0.034x - 5.309\mathrm{E}^{-6}x^2 - 2.468\mathrm{E}^{-7}x^3$	0.98843
Exponential fit	$y = \exp(0.224 + 0.016x + -4.163\mathrm{E}^{-5}x^2)$	0.98428

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