NUCLEAR THEORY, Vol. 41 (2024)
eds. M. Gaidarov, N. Minkov, Heron Press, Sofia

S-Matrix Analysis of High Energy Elastic Alpha Scattering on the ^{58}Ni Target

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Abstract. Elastic scattering of α-particles on medium-weight nuclei ^{58}Ni is analyzed at energies from 26 to about 175 MeV/nucleon using the original 6-parameter *S*-matrix model. Energy dependences of the total reaction cross-sections, nuclear rainbow angles, Fraunhofer crossover angles, and the model parameters are studied. The behavior of the quantum deflection function and the *S*-matrix modulus are investigated as well. The smooth behavior of the studied quantities and acceptable values of χ^2/N obtained in calculations by the *S*-matrix model demonstrate the applicability of this model in the entire considered energy range. The results are compared to the ones obtained from the optical model.

1 Introduction

In the present paper, we investigate the characteristic features of refraction phenomena observed in the elastic scattering of α-particles on ^{58}Ni nuclei at energies up to 175 MeV/nucleon [1]. The diffraction and refraction phenomena, as well as the interaction mechanisms of α-particles with the ^{58}Ni nucleus, are analyzed using the *S*-matrix approach in a wide range of energies (from 104 to 699 MeV). The study employs the original *S*-matrix model [2] and continuing series of researches [3–5]. By fitting the experimental data on the elastic scattering of α-particles on ^{58}Ni nuclei, we obtain the scattering matrix parameters for this *S*-matrix model. The study investigates the cross-section behavior in the region of the Fraunhofer crossover (where the near and far components of the scattering amplitude intersect) and the angles of the nuclear rainbow across the considered wide range of energies for the α-^{58}Ni elastic scattering. In order to assess the quality of the fitting of the data, which has been obtained using the *S*-matrix model, we also have considered the results of the analysis of the elastic α-^{58}Ni scattering using the optical-model (both the phenomenological and double-folding one) from literature [2, 6, 7].

The investigation covers the theoretical analysis of the results of several experiments where the differential cross-sections for the elastic α-^{58}Ni scattering were obtained at different alpha-particle energies E_α, namely: 104 [8], 139 [9], 172.5 [10], 240 [11], 288 [12], 340 [12], 386 [13], 480 [12], 699 [12] MeV. In addition to the description of the nuclear rainbow phenomenon in elastic α-^{58}Ni scattering, this article also examines the behavior of the Fraunhofer crossover angle and the nuclear rainbow angle with energy change, as well as simultaneous changes in the shapes of the quantum deflection function and scattering matrix modulus as functions of angular momentum. The study also considers the behavior of several parameters as functions of energy, including the nuclear refraction parameter, the nuclear transparency coefficient, the boundary radii and the nuclear-surface diffuseness parameters for the regions of strong absorption and nuclear refraction, as well as the energy dependence of the total reaction cross section. The results reveal smooth, physically plausible variations in these parameters with energy, which can be clearly explained in a semiclassical interpretation.

2 Theoretical Framework

In the phenomenological S-matrix model, the interaction between colliding nuclei is described directly by the S-matrix, which relates the initial and final states of the system and contains all needed information about the considered scattering process. In the present study, we use the Faxen-Holtsmark formula to express the elastic scattering amplitude with the S-matrix elements

$$f(\theta) = \sum_{l=0}^{\infty}(2l+1)(S_l - 1)P_l(\cos\theta), \quad (1)$$

where θ is the scattering angle, l is the angular momentum, S_l are the diagonal matrix elements of the scattering matrix in the representation of angular momentum, and $P_l(\cos\theta)$ are the Legendre polynomials. The following original parametrization of the S-matrix as a function of the angular momentum is used [2]:

$$S(L) = \eta(L)\exp(2i[\delta_r(L) + \sigma_c(L)]), \quad (2)$$

$$\eta(L) = \exp[\ln(\varepsilon)g(L, L_0, \Delta_0)], \quad (3)$$

$$2\delta_r(L) = \delta_1 g^2(L, L_1, \Delta_1), \quad (4)$$

$$g(L, L_j, \Delta_j) = \frac{\sinh(L_j/\Delta_j)}{\cosh(L_j/\Delta_j) + \cosh(L/\Delta_j)}, \quad j = 0.1 \quad (5)$$

where $L = l + 1/2$, $\eta(L)$ is the modulus of the scattering matrix, δ_r and $\sigma_c(L)$ are the nuclear (refractive) and Coulomb phase shifts, $\varepsilon \ll 1$ is a parameter which defines the nuclear matter transparency, δ_1 is the parameter which characterizes the magnitude of nuclear refraction of the scattered waves, L_j and Δ_j are the parameters which define the linear dimensions and diffuseness of the strong absorption ($j = 0$) and nuclear refraction ($j = 1$) regions. Therefore, the chosen model contains 6 free parameters: δ_1, ε, L_0, Δ_0, L_1, and Δ_1, which are to be determined from the fitting procedure. We define the Coulomb phase shift $\sigma_c(L)$ as the phase of the point-like charge scattering by the uniformly charged sphere having the radius R_c. The explicit expression can be found in Ref. [2]. The nuclear rainbow angle θ_r is defined as the depth of the minimum of the quantum deflection function (7).

The differential cross-section of elastic scattering is calculated from the amplitude (1) according to

$$\sigma(\theta) = \frac{d\sigma}{d\Omega} = |f(\theta)|^2. \tag{6}$$

The quantum deflection function $\theta(L)$ is related to the scattering phase $\delta(L) = \delta_r(L) + \sigma_c(L)$ by the formula

$$\theta(L) = 2\frac{d\delta(L)}{dL}. \tag{7}$$

The parameters of the model have been determined by minimizing the standard χ^2/N value per an experimental point, using a gradient method, to achieve the best agreement of the calculated elastic scattering cross-section with the experimental data. A uniform experimental error of 10% was assumed for all the data considered. The complex structures observed in the elastic scattering cross-sections, such as the Fraunhofer crossover, have been analyzed using the Fuller procedure [14], decomposing these cross-sections into the near- and far-side components, which was somewhat refined to remove certain fictitious contributions, as described in Ref. [2].

3 Results of Analysis of the Elastic α-^{58}Ni Scattering

We present the results of the analysis of the data available for the elastic scattering of ^{4}He ions on the ^{58}Ni nucleus in the energy range E_α = 104699 MeV, which aims to examine the manifestations of the nuclear rainbow and other refraction effects over this wide range of α-particle energies.

We have obtained the S-matrix parameters for the studied scattering cases by analyzing the experimental data available (9 data sets) for the α-^{58}Ni elastic scattering. The differential cross-sections of the α-^{58}Ni elastic scattering, calculated for the picked values of energy based on the S-matrix parametrization in the form (2)-(5), are given in Figures 1–4, other five can be found in Ref. [1]. When calculating the scattering amplitude, the sum in formula (1) was taken for a wide enough range of l values to ensure sufficient accuracy of the calculated cross-sections. The S-matrix parameters have been found by fitting them to the corresponding experimental data, and further, their dependence on the energy has been analyzed (see Table 1). In the figures, for comparison, we also

Table 1. Energy evolution of the S-matrix parameters, the total reaction cross-section σ_r, the nuclear rainbow angle θ_r, the Fraunhofer crossover angle $\theta_c r$, and the χ^2/N values for the calculated elastic scattering cross-sections (the values with asterisk were taken from the experimental data analysis by the optical model)

E, MeV	104	139	172.5	240	288	340	386	480	699
k, fm^{-1}	4.20	4.86	5.42	6.42	7.05	7.67	8.20	9.18	11.2
L_0	24.1	27.5	30.0	33.2	38.6	38.6	46.4	46.6	66.1
L_1	19.9	24.4	27.0	26.2	38.0	39.9	41.8	49.0	64.2
Δ_0	2.57	3.53	3.67	4.68	3.92	6.00	7.46	6.38	21.76
Δ_1	5.10	5.79	6.34	9.91	7.58	9.61	9.73	18.2	11.9
$\varepsilon \times 100$	2.41	2.54	3.12	2.98	8.26	4.56	13.2	5.39	45.3
δ_1	27.3	21.3	18.9	23.0	9.82	9.50	8.84	7.31	3.49
χ_S^2/N	1.5	2.0	1.2	2.7	2	1.1	1.1	4.4	6.9
χ_{opt}^2/N	5.4*	6.2	2.8*	-	4.0*	2.6*	-	2.6*	5*
σ_r, mb	1662	1760	1616	1526	1368	1466	1663	1383	1651
θ_r	78.7	53.9	43.9	33.1	18.2	13.5	12.5	4.2	3.5
θ_{cr}	15.0	12.0	9.0	7.0	6.0	4.9	4.8	2.5	2.6

present the differential cross-sections calculated based on the optical model. In Table 1, the values of χ^2/N_{optic} are also presented for comparison. The analyzed cross-sections exhibit pronounced refractive scattering patterns (as shown in Figures 1–4). Generally, for the scattering of ^{4}He on the medium nucleus ^{58}Ni, a

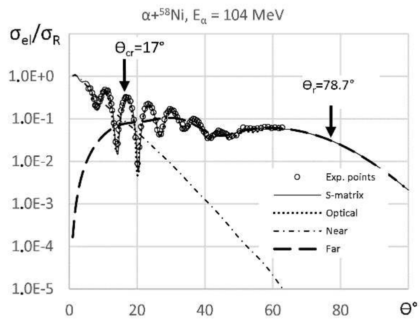

Figure 1. Differential cross-section (ratio to the Rutherford one) of the elastic $\alpha-^{58}$Ni scattering at $E_\alpha = 104$ MeV calculated in the S matrix model (2)-(5) (solid line), its far- (dashed line) and near-side (dash-dotted line) components; dotted line is for the optical model. The experimental data are from Ref. [8]. Arrows show the rainbow and crossover angles.

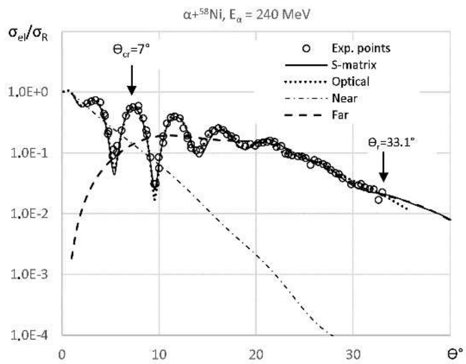

Figure 2. The same as in Figure 1 but for $E_\alpha = 240$ MeV. The experimental data are from Ref. [11].

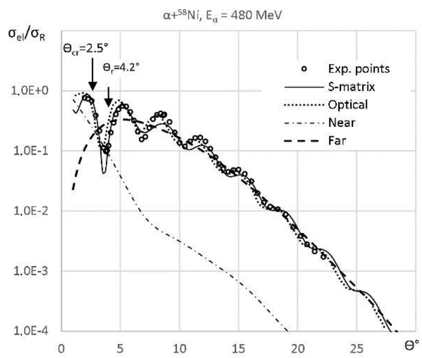

Figure 3. The same as in Figure 1 but for $E_\alpha = 480$ MeV. The experimental data are from Ref. [12].

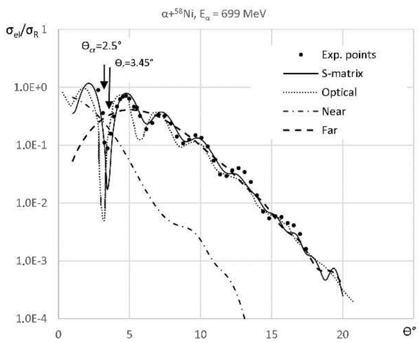

Figure 4. The same as in Figure 1 but for $E_\alpha = 699$ MeV. The experimental data are from Ref. [12].

distinct manifestation of the nuclear rainbow scattering is observed at all considered energies, as evidenced by the characteristic shape of the found deflection functions. The deflection function, denoted by $\theta(L)$, displays an approximately symmetrical shape about its minimum in the region of negative values, which is an indication of the nuclear rainbow effect.

Based on Figures 1–4, it can be stated that the proposed parametrization of the scattering matrix makes it possible to accurately describe the complex behavior of the differential cross-sections of elastic scattering of α-particles by ^{58}Ni nuclei over wide ranges of energies and scattering angles. The experimentally measured cross-sections exhibit multiple Fraunhofer oscillations at small scattering angles, followed by a decrease in the amplitude of the oscillations and the emergence of a broad maximum, indicating the presence of nuclear rainbow scattering. Besides, the S-matrix model shows sufficiently accurate results in comparison with the optical model (see the figures and the columns of Table 1 with χ^2/N).

Let us examine the behavior of the S-matrix as a function of the angular momentum L depending on the projectile energy. Figures 5 and 6 present the quantum deflection functions $\theta(L/k)$ and the modulus $\eta(L/k)$ versus the quasi-classical impact parameter L/k for all scattering cases under consideration. It is seen from Figure 5 that the quantum deflection function $\theta(L)$ displays a minimum in the region of its negative values at all energies, which is a necessary condition for the manifestation of the nuclear rainbow effect. As the projectile energy rises, the depth of this minimum decreases considerably. Therefore, based on Figure 5 and Table 1, it can be concluded that the nuclear rainbow angle θ_r decreases with increasing energy. Figure 6 shows that the function $\eta(L)$ has the form of a "step" with a diffuse surface near $L \approx L_0$ where $\eta(L)$ changes rapidly from the value of nuclear transparency ε up to unity. This corresponds to the presence of strong absorption of incident α-particles by the ^{58}Ni nucleus at $L < L_0$.

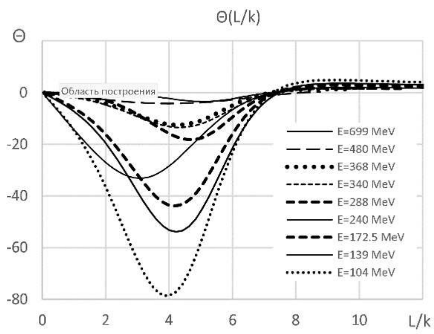

Figure 5. Quantum deflection function $\theta(L/k)$ (degrees) as a function of the impact parameter L/k (fm) for different energies.

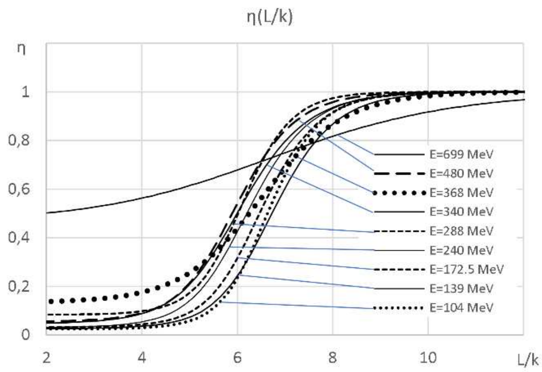

Figure 6. Dependence of the scattering matrix modulus $\eta(L/k)$ on the impact parameter L/k (fm) for different energies.

Considering the values of parameters presented in Table 1 and Figures 7-9, one can observe their evolution with energy. The boundary radii L_0/k and L_1/k, and the diffuseness parameters Δ_0/k and Δ_1/k follow approximately linear behavior with a quite low slope angle. The nuclear matter transparency coefficient ε gradually increases with (presumably according to an approximately linear law) with the energy increase. The magnitude of the nuclear scattering

(a)

(b)

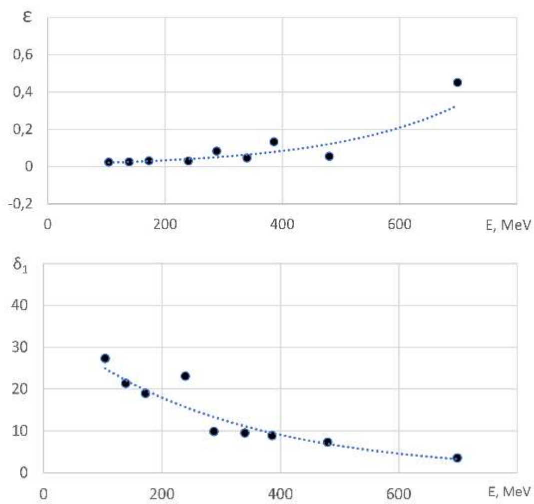

Figure 7. The nuclear matter transparency coefficient ε (a) and the magnitude of nuclear refraction δ_1 (b), obtained from the data analysis by the S-matrix parametrization (2)–(5) for the α-^{58}Ni elastic scattering depending on energy (points). Dotted lines show the suggested approximating law.

(a)

(b)

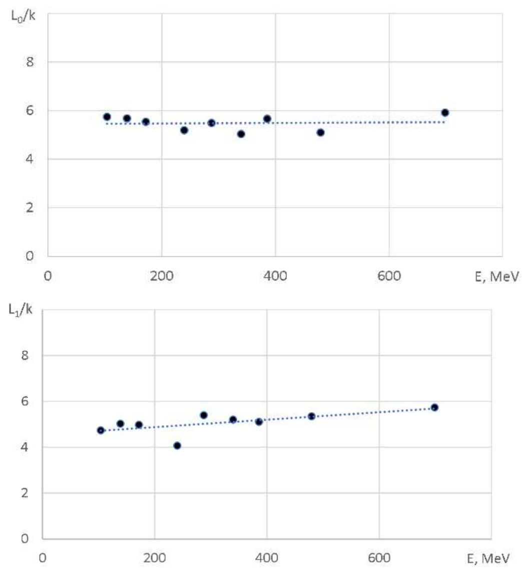

Figure 8. Dependence of (a) the boundary radii L_0/k and (b) L_1/k (fm) on the energy. Dotted lines show the suggested approximating law.

(a)

(b)

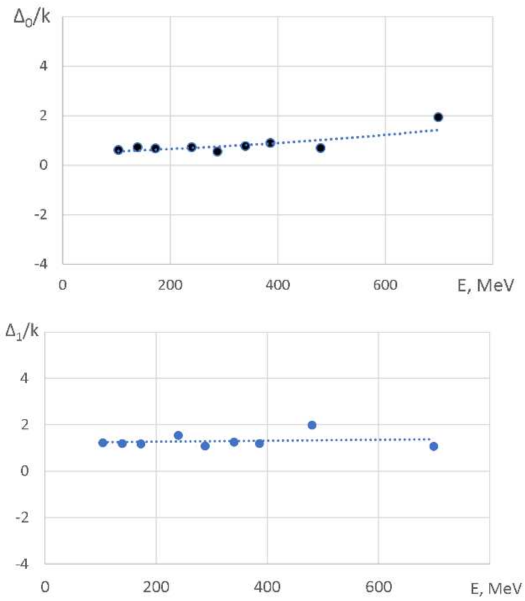

Figure 9. Dependence of the diffuseness parameters: (a) Δ_0/k; and (b) Δ_1/k (fm) on the energy. Dotted lines show the suggested approximating law.

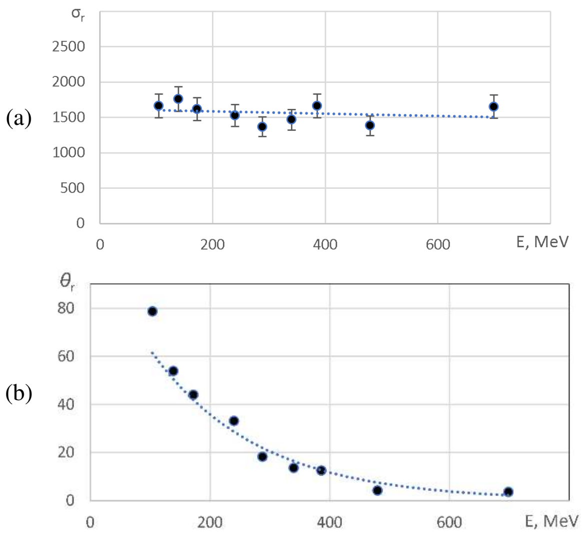

Figure 10. The total reaction cross-section σ_r (mb) (a); and the nuclear rainbow angle θ_r (b), obtained from the data analysis by the S-matrix model for the α-^{58}Ni elastic scattering depending on the energy (points). Error bars correspond to an assumed 10% uncertainty. Dotted lines show the suggested approximating law.

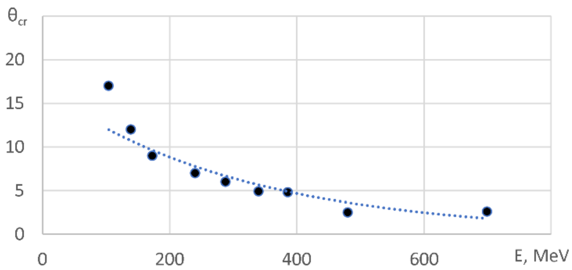

Figure 11. Angle of the Fraunhofer crossover θ_{cr}, obtained from the data analysis by the S-matrix model for the α-^{58}Ni elastic scattering as a function of energy (points). Dotted line shows the suggested approximating law.

phase δ_1 decreases approximately according to an inverse exponent law $\delta_1 \sim \exp(-aE_\alpha)$, where $a = 0.003$ MeV^{-1}. The dependence of total reaction cross-section σ_r, nuclear rainbow angle θ_r and the angle of the Fraunhofer crossover θ_{cr} are shown in Figure 10 (a,b) and Figure 11, respectively.

4 Conclusion

The presented analysis reveals that the earlier proposed original six-parameter model of the scattering matrix effectively accounts for the behavior of the differential cross-sections of the elastic scattering of α-particles by medium ^{58}Ni nuclei over wide ranges of scattering angles and energies (104-699 MeV). This model correctly describes the observed effects of both nuclear diffraction and refraction in the scattering processes under study. The differential cross-sections of the elastic α-^{58}Ni scattering have exhibited the existence of the nuclear rainbow effect at all examined energies within the range of 26175 MeV/nucleon. By fitting the studied experimental data on α-^{58}Ni elastic scattering, we have determined the values of the parameters of the used model and have studied the behavior of the scattering matrix. Additionally, we have studied the energy systematics of the nuclear rainbow angle θ_r and the angle of Fraunhofer crossover θ_{cr} of the near- and far-side components in the angular distribution of cross-sections. We have also investigated the behavior of the shapes of the quantum deflection function $\Theta(L)$ and the modulus of the S-matrix $\eta(L)$ depending on energy. It has been found that the deflection function retains a nearly symmetrical profile in the region of negative values relative to its rainbow minimum as the energy increases. Furthermore, the nuclear rainbow angle θ_r and the angle of Fraunhofer crossover θ_{cr} demonstrate an exponential decrease, $\exp(-bE_\alpha)$, with increasing energy. Also, we have analyzed the behavior of parameters of the S-matrix and their dependencies.

A comparative examination of the accuracy of fitting data in the S-matrix and optical-model approaches reveals the high quality of fitting achieved using the proposed six-parameter S-matrix model. This result suggests that the considered S-matrix approach can be used to study a wide range of refraction phenomena observed for light-ion scattering on medium and heavy target nuclei, and it can also provide additional useful insights into the investigation of the interaction between nuclei and nuclear structure study.

Acknowledgements

These proceedings are presented in memory of our colleagues Berezhnoy Yu.A. and Pylypenko V.V. with sincere gratitude for their many years of joint fruitful work in the field of theoretical nuclear and high energy physics. The project was funded and supported by the Ukrainian government grants: 0121U109812, 0122U001303, 0122U001483, 0122U001483, 0119U002523 and by the US National Science Foundation (NSF) IMPRESS-U grant 2403609 via STCU project 7120.

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