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Superscaling Analysis of Inclusive Electron Scattering within the Coherent Density Fluctuation Model

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Abstract. The experimental data from quasielastic electron scattering on ^{12}C are reanalyzed within a theoretical method in terms of a new scaling variable ψ^* suggested by the interacting relativistic Fermi gas model with scalar and vector interactions, which is known to generate a relativistic effective mass for the interacting nucleons. We use a new scaling function $f^{\rm QE}(\psi^*)$ for the inclusive lepton scattering from nuclei within the coherent density fluctuation model (CDFM). The latter is a natural extension of the relativistic Fermi gas (RFG) model to finite nuclei. In this work, on the basis of the scaling function obtained within CDFM with a relativistic effective mass $m_N^* = 0.8m_N$, we calculate and compare the theoretical results with a large set of experimental data for inclusive (e,e') cross sections. The model explicitly considers the modification of the relativistic effective mass of the nucleon within the relativistic mean field model of nuclear matter. In our calculations is incorporated the contribution of electroweak two-body currents in the two-particle two-hole sector, which are consistently computed within the same model. Good agreement with experimental data is found over the whole range of electron energies.

1 Introduction

The superscaling phenomenon was firstly considered within the framework of the Relativistic Fermi Gas (RFG) model [1–6], where a properly defined function of the scaling ψ-variable was introduced. At large transferred momentum $q = |\mathbf{q}|$ ($q > 500$ MeV/c) the latter does not depend on q and the mass number. As pointed out in [4], however, the actual nuclear dynamical content of the superscaling is more complex than that provided by the RFG model. It was observed that the experimental data have a superscaling behavior in the low-ω side (ω being the transfer energy) of the quasielastic peak for large negative values of ψ (up to $\psi \approx -2$), while the predictions of the RFG model are $f(\psi) = 0$ for $\psi \leq -1$. This imposes the consideration of the superscaling in realistic finite systems. One of the approaches to do this was developed [7, 8] in the

CDFM [9–16] which is related to the δ-function limit of the generator coordinate method [7, 17]. It was shown in [7, 8, 18] that the superscaling in nuclei can be explained quantitatively on the basis of the similar behavior of the high-momentum components of the nucleon momentum distribution in light, medium and heavy nuclei. It is well known that the latter is related to the effects of the *NN* correlations in nuclei (see, *e.g.* [9, 10]).

In our previous works [7, 8, 18, 19] we obtained the CDFM scaling function $f(\psi)$ starting from the RFG model scaling function $f_{\text{RFG}}(\psi)$ and convoluting it with the weight function $|F(x)|^2$ that is related equivalently to either the density $\rho(r)$ or the nucleon momentum distribution $n(k)$ in nuclei. Thus, the CDFM scaling function is an infinite superposition of weighted RFG scaling functions. This approach improves upon RFG and enables one to describe the scaling function for realistic finite nuclear systems. The CDFM scaling function has been used to predict cross sections for several processes such as the inclusive electron scattering in the QE and Δ- regions [19, 20] and neutrino (antineutrino) scattering both for charge-changing (CC) [20] and for neutral-current (NC) [21] processes. In our work [19] we reproduce experimental data of the inclusive electron scattering in the QE-region using CDFM scaling function which is obtained by the parameterizing the RFG scaling function and by the coefficient c_1, which helps us to account for the experimental fact of the asymmetry of the scaling function. The value of the coefficient c_1 ($c_1 \neq 3/4$) is taken in accordance with the empirical data (c_1 depends on the value of the momentum transfer in the QE peak).

In the present work we follow Ref. [22], where the ψ^* scaling idea is explored in the context of the Relativistic Mean Field (RMF) for nuclear matter. The new scaling function $f^*(\psi^*)$ including dynamical relativistic effects [22–25] is introduced through an effective mass into its definition. The resulting superscaling approach with relativistic effective mass (SuSAM*) model describes a large amount of the electron scattering data lying inside a phenomenological quasielastic band, and it has been extended recently successfully to the neutrino and antineutrino sector [26] giving a fair agreement with the data. An enhancement of the SuSAM* model is detailed in Refs. [27–30], where the responses of 2p2h meson exchange currents (MEC) are calculated consistently with the mean field model in nuclear matter. This is achieved by incorporating an effective mass and vector energy for the nucleon, thereby explicitly including the same medium modifications as the quasielastic responses. A systematic review of experimental data on quasielastic neutrino scattering reveals a reasonable agreement with the theoretical predictions derived from the extended SuSAM* model [30].

The SuSAM* model was first developed using the set of ^{12}C data [22, 23] and later applied to other nuclei in [24]. In Ref. [22] was obtained the best value of the effective mass $M^* = m_N^*/m_N = 0.8$, which we use in our present consideration. This value provides the best scaling behavior of the data with a large fraction of data concentrated around the universal scaling function of the relativistic Fermi gas

$$f_{\mathrm{RFG}}(\psi^*) = \frac{3}{4}(1-\psi^{*2})\theta(1-\psi^{*2}). \qquad (1)$$

The ψ^* variable was inspired by the mean-field theory, that provides a reasonable description of the quasielastic response function [31, 32]. The important point

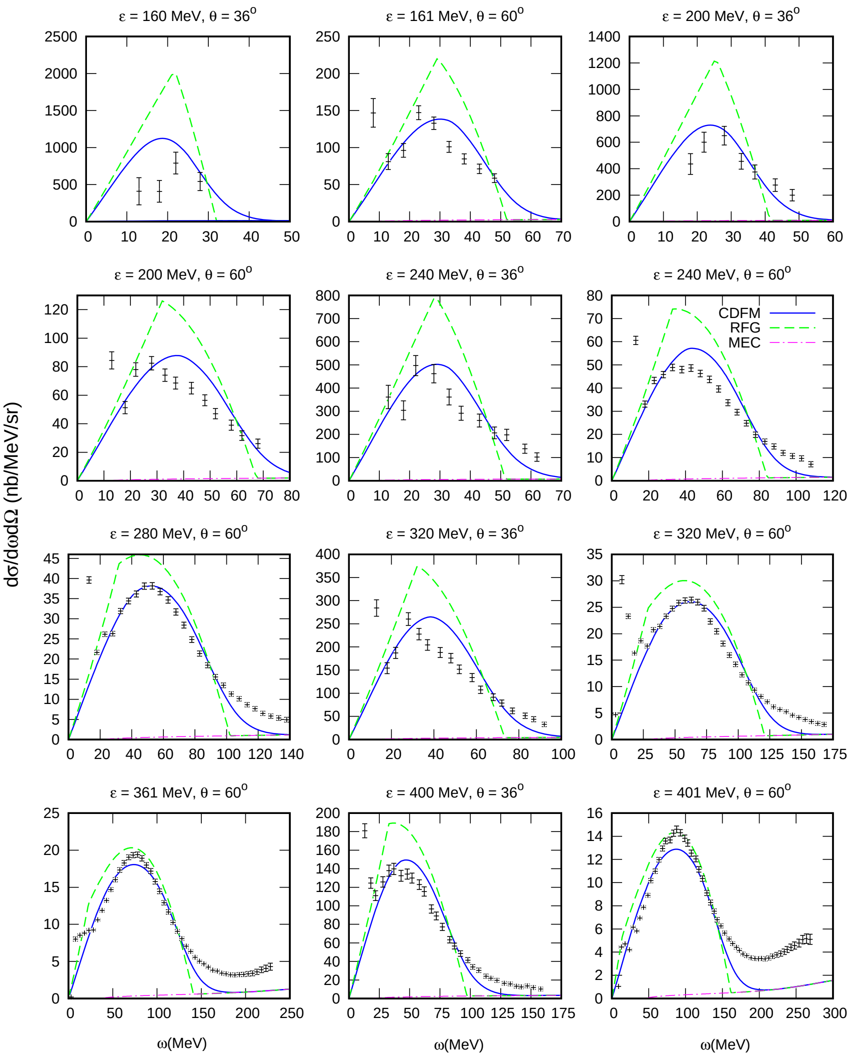

Figure 1. (Color online) The CDFM$_{M^*}$ results for the inclusive (e, e') cross section for several kinematics compared to the RFG$_{M^*}$ model and experimental data.

is that in the interacting RFG model the vector and scalar potentials generate an effective mass m_N^* for the nucleon in the medium.

Our present approach, called CDFM$_{M^*}$ (CDFM with M^*), uses scaling function obtained within the CDFM model. It keeps the gauge invariance and describes the dynamical enhancement of both the lower components of the relativistic spinors and the transverse response function. In Ref. [33], we thoroughly examined the theoretical framework for deriving the CDFM$_{M^*}$ scaling function

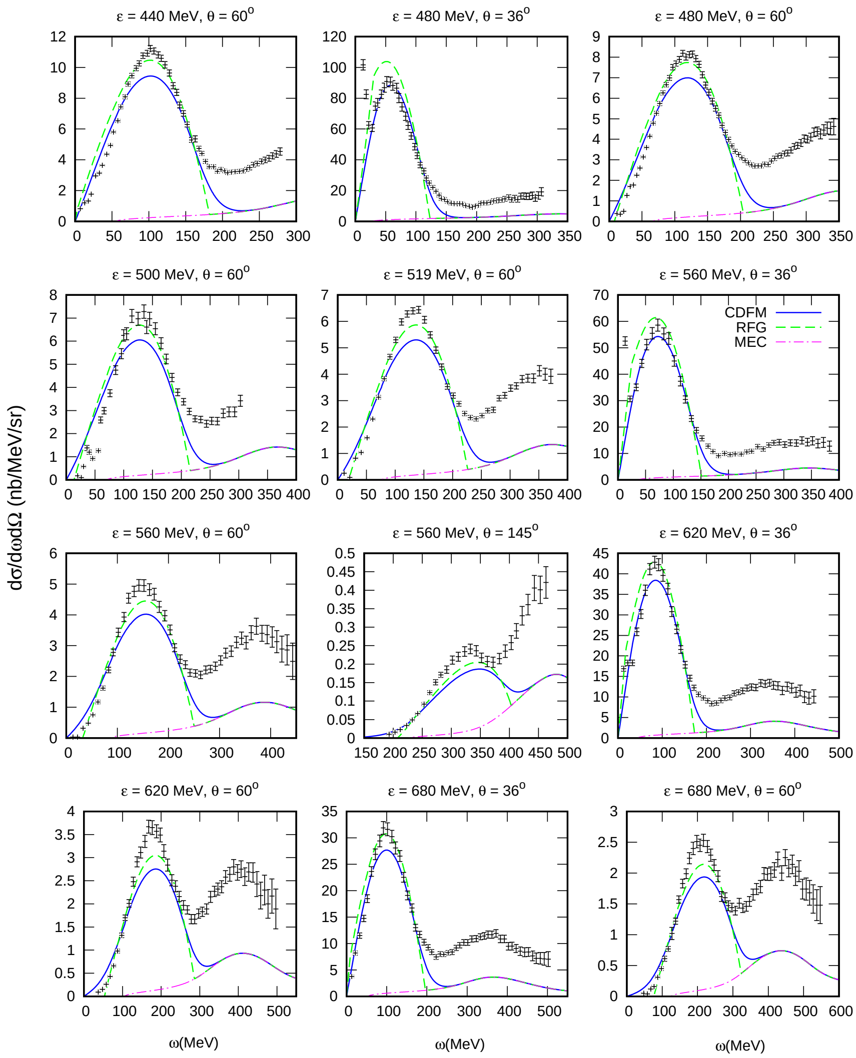

Figure 2. (Color online) The CDFM$_{M^*}$ results for the inclusive (e, e') cross section for several kinematics compared to the RFG$_{M^*}$ model and experimental data.

and the general formalism for characterizing the (e, e') and (anti)neutrino CC quasielastic double differential cross sections. In this research, in contrast to our earlier study [33], the responses of the 2p2h MEC are computed in accordance with the mean field model within nuclear matter, and we provide our results for the inclusive (e, e') cross sections utilizing the CDFM$_{M^*}$ approach.

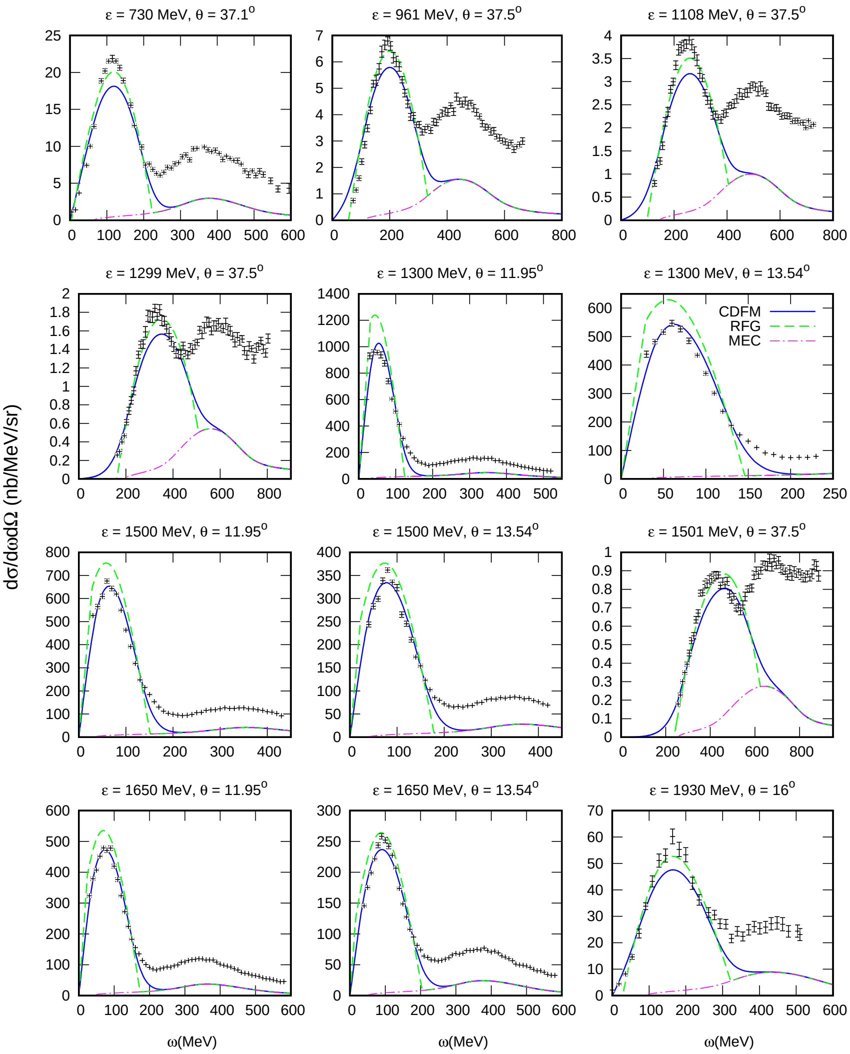

Figure 3. (Color online) The CDFM$_{M^*}$ results for the inclusive (e, e') cross section for several kinematics compared to the RFG$_{M^*}$ model and experimental data.

2 Results and Discussions

In this section we use the new scaling function of the CDFM$_{M^*}$ model (see Ref. [33]) to compute lepton scattering cross sections on ^{12}C. It is important to test CDFM$_{M^*}$ model for inclusive (e, e') scattering before to apply it to neutrino scattering. In Figures 1–4 we show the predictions of CDFM$_{M^*}$+MEC contribution (blue solid line) for the (e, e') cross section compared to the experimental

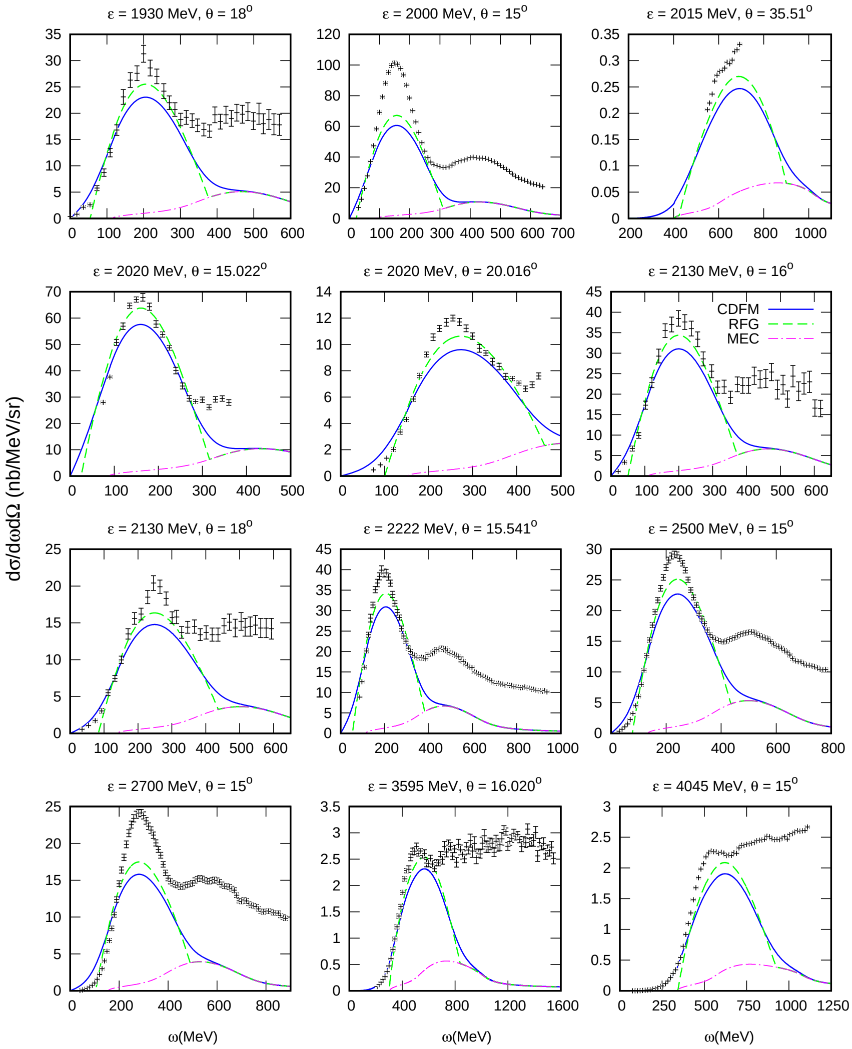

Figure 4. (Color online) The CDFM$_{M^*}$ results for the inclusive (e, e') cross section for several kinematics compared to the RFG$_{M^*}$ model and experimental data.

data [34]. Also, the RFG$_{M^*}$+MEC results (green dashed line) are given. The contribution of meson exchange currents (MEC) is presented, separately. The CDFM$_{M^*}$ model description is quite acceptable using just one free parameter, namely the effective mass M^*, which is fixed to 0.8 in all performed calculations.

It is important to recognize that the parametrization of the 2p2h MEC used in our analysis [33] originates from the RFG model with an effective mass of $M^* = 1$. An alternative parametrization for electroweak 2p2h MEC responses, computed in the RMF with an effective mass of $M^* = 0.8$, has been proposed in recent studies [27, 28]. This new parametrization, utilizes a semiempirical formula for two-nucleon emission responses and is fitted to the exact results for momenta in the range $q = 200$ and 2000 MeV/c. The semiempirical formula is used in the present research and allows us to compute accurately the 2p2h MEC responses using an analytical formula, which reduces the calculation time.

Table 1. Values of initial electron energy ϵ, scattering angle θ, and transferred momenta q_{exp}^{QE} for the cases of inclusive electron scattering cross sections considered

Figures 1 and 2			Figures 3 and 4		
ϵ [MeV]	θ [deg]	$\approx q_{exp}^{QE}$ [MeV/c]	ϵ [MeV]	θ [deg]	$\approx q_{exp}^{QE}$ [MeV/c]
160	36.00	94	730	37.10	442
161	60.00	151	961	37.50	585
200	36.00	118	1108	37.50	675
200	60.00	188	1299	37.50	793
240	36.00	143	1300	11.95	269
240	60.00	225	1300	13.54	305
280	60.00	258	1500	11.95	312
320	36.00	191	1500	13.54	353
320	60.00	294	1501	37.50	922
361	60.00	329	1650	11.95	343
400	36.00	238	1650	13.54	389
401	60.00	365	1930	16.00	539
440	60.00	399	1930	18.00	606
480	36.00	285	2000	15.00	524
480	60.00	435	2015	35.51	1213
500	60.00	448	2020	15.02	532
519	60.00	465	2020	20.02	704
560	36.00	331	2130	16.00	599
560	60.00	504	2130	18.00	673
560	145.00	642	2222	15.54	605
620	36.00	366	2500	15.00	663
620	60.00	555	2700	15.00	726
680	36.00	401	3595	16.02	1326
680	60.00	602	4045	15.00	1122

For completeness, in Table 1 we list the initial electron energy, the scattering angle, as well as the approximate values of the transfer momentum q^{QE}_{exp} in the position of the maximum of the QE peak (ω^{QE}_{exp}) for all cases presented in Figures 1–4. From Figures 1–4, it can be observed that the CDFM$_{M^*}$ model effectively describes the experimental data, not only in the regime of high momentum transfer but also in the region of very low momentum transfer.

3 Conclusions

It is shown in our work that the CDFM$_{M^*}$ model describes successfully inclusive (e, e') quasielastic cross section on the basis of the new scaling variable ψ^*, of the empirical density distribution of protons used to determine the weight function $|F(x)|^2$, and of the corresponding scaling function $f^{\rm QE}(\psi^*)$. We note that in the CDFM$_{M^*}$ model an effective mass $M^* = m^*_N/m_N = 0.8$ is used. The latter is originating from the interacting RFG model in which the vector and scalar potentials generate the effective mass of the nucleon in medium. We should emphasize that the CDFM$_{M^*}$ scaling function keeps the gauge invariance (that is not the case in the SuSA approach) and describes the dynamical enhancement of the lower components of the relativistic spinors, as well as the transverse response function. In addition, we note the important fact that in the CDFM$_{M^*}$ model the weight and scaling functions are normalized to unity. It is pointed out that the constructed realistic CDFM$_{M^*}$ scaling function is an essential ingredient in this approach for the description of the processes of lepton scattering from nuclei. The successful test of the CDFM$_{M^*}$ model for inclusive (e, e') scattering gives us the confidence to extend its application to neutrino scattering.

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