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Vector Boson Model Application with SU(3)-irrep Dependent Effective Charges

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Abstract. The Vector Boson Model (VBM) with broken SU(3) symmetry is implemented for calculation of nuclear ground- and γ- band levels with direct description of B(E2) transition values. By examining the original model B(E2) values calculated for ^{166}Er in different SU(3) irreducible representations (irreps), we found that relevant scaling factors (effective charges) depending on the SU(3)-irrep quantum numbers (λ, μ) can be introduced, so that the accordingly normalized B(E2)s become directly comparable with experimental data, without need to consider dimensionless transition ratios. The approach is numerically tested on the ground-γ band spectrum of ^{166}Er up to angular momentum $L = 16$ and the corresponding most favourable SU(3)-irrep providing the best model description is determined. The possibility for VBM application with the use of proxy-SU(3) irreps is pointed out.

1 Introduction

Some specific properties of collective spectra developed in heavy deformed nuclei can be understood on a symmetry basis. One of the pioneering approaches in this direction employs the SU(3) symmetry implemented in terms of vector boson excitations, the Hamiltonian of which generates the energy spectrum by reducing SU(3) to the rotation symmetry O(3) [1]. In this vector boson model (VBM) the ground- and γ- rotation bands of a deformed even-even nucleus are described in the form of a split SU(3)-multiplet determined by the quantum numbers (λ, μ) of given irreducible representation (irrep) of the group SU(3). A numerical realization of the model allowing calculation in any (λ, μ)-irrep with λ, μ even and $\lambda > \mu$ is elaborated in [2]. No model restrictions on the possible SU(3)-irreps used to describe the ground-γ band spectrum were imposed, except for the lower $\lambda \geq L_{\max}$ limit ensuring that the states with the highest angular momentum $L_{\max}$ are accommodated in the multiplet. Due to the scarce data on experimental B(E2) transition probabilities at the time, certain ratios

between B(E2)-values, allowing comparison to additional experimental information, were introduced in [1] and later adopted in [2]. In this framework VBM calculations were made for the ground-γ band spectra of several rare-earth and actinide nuclei up to angular momentum $L = 8$ [2]. For each nucleus a variety of (λ, μ)-irreps were tested and the corresponding favored (λ, μ)-irrep, providing the best model description was determined.

Presently a question arises about the applicability of VBM towards higher angular momenta, given that current data on the ground- and γ- bands reach $L \approx 20$ and even more [3]. Since a wealth of data on B(E2)-values is also available, it makes now sense to directly compare the theoretical B(E2)-values (in W.u units) with the corresponding data instead of using ratios of B(E2)s.

To clarify the above question, in this work we extend the VBM framework towards higher angular momenta up to $L = 16$. We also introduce specific scaling factors (effective charges) depending on the (λ, μ)-numbers which allow us to directly compare the model B(E2)-values with the data without using dimensionless ratios. We then test the modified algorithm on the ground-γ band spectrum of the nucleus ^{166}Er and examine the way in which the corresponding favored (λ, μ)-irrep can be determined. We also test the model on the SU(3)-irrep determined for this nucleus within the proxy SU(3) symmetry scheme on the basis of its intrinsic shell structure [4, 5].

In Sec. 2 the determination of the (λ, μ)-dependent effective charges is presented. In Sec. 3 VBM calculations made for ^{166}Er with the effective charges and determination of the effectively most favored (EMF) SU(3)-irrep are presented. The result for the relevant proxy SU(3) irrep is also given. In Sec. 4 concluding remarks are given.

2 SU(3) irrep Dependence and Scaling of the VBM Transition Probabilities

The VBM formalism and its numerical realization are comprehensively presented in Ref. [2] while the proxy-SU(3) scheme is originally formulated and presented in Refs. [4, 5]. In the present work the original VBM model algorithm and the corresponding computer code are modified so as now the calculated ground- and γ- intraband and interband B(E2) transition probabilities can be directly compared to the corresponding experimental data in the model adjustment procedure. This modification replaces the original version in which several ratios between B(E2) transitions were adjusted (see Eqs. (20) and (21) in Ref. [2]).

The present calculations are made by the modified version of the VBM code by fitting the full set of experimental gsb and γ-band energy levels and the attendant B(E2) transition values of ^{166}Er up to $L = 16$ taken from [3]. The aim is to determine appropriate scaling factors (effective charges) allowing us to obtain unambiguous model descriptions or predictions along the different (intra- and interband) transition paths.

At the first step the B(E2)-values were directly calculated using the VBM model expression of Eq. (19) in [2]. The calculations were made for the SU(3) irreps: $(\lambda = 16,\ \mu = 2, 4, \ldots, 14)$; $(\lambda = 52,\ \mu = 2, 4, \ldots, 14)$ and $(\lambda = 30,\ \mu = 2, 4, \ldots, 14)$. The series of tested irreps include the (16,2) irrep determined in [2] as "favored" with respect to the lowest, up to angular momentum $L = 8 - 10$, energy levels and transition values (see Table II in [2]). The (52,14) irrep is attributed to ^{166}Er in [5] (Table II therein) on the basis of the proxy SU(3) scheme. The irreps with μ between 2 and 14, as well as, those with the intermediate $\lambda = 30$ are taken to examine the SU(3)-irrep dependence of the VBM model description of ^{166}Er. To be comparable, the calculations for the different (λ, μ) irreps were performed under the same numerical conditions including initial values of the adjusted parameters, weights of the B(E2)-values, etc.

The result of these calculations is shown in Table 1 where for each considered (λ, μ) irrep we give the root-mean-square (RMS) deviations between the theoretical and experimental energy levels and B(E2) transition values, σ_E and σ_B, respectively, determined as

$$\sigma_E = \sqrt{\frac{1}{n_E}\sum_{\nu=1}^{n_E}\left[E_\nu^{\rm th} - E_\nu^{\rm exp}\right]^2},\quad \sigma_B = \sqrt{\frac{1}{n_B}\sum_{\nu=1}^{n_B}\left[B(E2)_\nu^{\rm th} - B(E2)_\nu^{\rm exp}\right]^2}, \tag{1}$$

where n_E and n_B are the total numbers of experimental energy and B(E2) values described, respectively.

The analysis of these results shows that the model fit with the favored $(\lambda, \mu) = (16, 2)$ irrep of Ref. [2] gives a good description of the energy levels and B(E2) transition values while the fit with the proxy-SU(3) irrep (52,14) [5] gives some larger energy RMS deviation σ_E and huge, by more than an order, deviation σ_B between theoretical and experimental B(E2)s. The systematic analysis of model descriptions obtained for the (λ, μ)-irreps between these two irreps, including $(\lambda = 30, \mu = 2, 4, \ldots 14)$, shows that with the distance from the (16,2)-irrep an overestimation of the gsb- and γ- intraband B(E2)-values increases with the

Table 1. Energy and B(E2) RMS deviations σ_E (in keV) and σ_B (in W.u.) obtained for various (λ, μ)-irreps in ^{166}Er without use of scaling factors in the B(E2)-values

λ	μ	σ_E	σ_B	λ	μ	σ_E	σ_B	λ	μ	σ_E	σ_B
16	2	57.674	38.782	30	2	60.954	593.529	52	2	65.796	2179.215
16	4	76.169	50.112	30	4	62.772	651.024	52	4	66.375	2273.434
16	6	70.519	81.042	30	6	65.755	712.053	52	6	67.312	2370.952
16	8	169.096	127.712	30	8	70.548	777.065	52	8	68.613	2471.913
16	10	375.453	185.438	30	10	78.117	846.748	52	10	70.361	2576.495
16	12	538.432	245.178	30	12	95.193	922.229	52	12	73.116	2684.930
16	14	692.485	309.237	30	14	132.573	1005.555	52	14	76.093	2797.531

increase of both λ and μ entailing corresponding increase in σ_B. At the same time an overestimation of the ground-γ interband B(E2)-values increasing with μ is observed whereas these values remain almost unaffected by the increase in λ. Also, in Table 1 we notice the essential increase of the energy RMS factor σ_E with the increasing of μ for λ=16 and to a less extend for λ=30. Generally this observation can be attributed to the fact that ^{166}Er is considered belonging to the region of well-deformed axially-symmetric nuclei in compliance with the favored minimal $\mu = 2$ value (supposing minimal triaxiality), whereas the structure of the SU(3) multiplets with large μ-values, especially such that $\mu/\lambda \gtrsim 0.5$, suppose essential triaxial deformation [6]. It should be noted that some recent studies suggest possible triaxiality in ^{166}Er and other well deformed nuclei [7,8]. Although in the end we shortly consider this point, we leave the more specific discussion on it for a subsequent work.

The finding that the "favored" SU(3)-irrep (16,2) directly reproduces the numerical scale of the experimental B(E2) transition values leads us to the idea to use it as a starting point for examining the SU(3)-irrep dependence of the model B(E2)-values and try to introduce corresponding irrep-dependent scaling factors (effective charges). It is important to note here that in the case this is the maximally symmetric SU(3)-irrep with minimal $\lambda = 16$ quantum number which accommodates the state with the maximal observed angular momentum $L = 16$ of the pair of ground- and γ-bands in ^{166}Er. (Note that the most symmetric SU(3)-irreps of the type $(\lambda, 0)$ can only accommodate one rotational band—the gsb one—and, therefore, they are irrelevant for the VBM scheme.) Based on the results in [2] one may consider that also for the other nuclei in the region of VBM applicability the "favored" SU(3)-irreps may be used in the same way. We, therefore, denote these irreps by (λ_0, μ_0).

Further, we examine in the calculation outputs the (λ, μ)-dependence of the ratio between theoretical and experimental B(E2) transition values, $B^{\text{th}}(E2)/B^{\text{exp}}(E2)$. By following it along the different SU(3) irreps and comparing with the ratio in the favored $(\lambda_0, \mu_0) = (16, 2)$ irrep, we find that the following scaling factors (squared effective charges) appropriately reduce the model B(E2)s keeping them within the same numerical scale fixed for given (λ, μ)-irrep:

$$e^2_{\text{eff-intra}} = \left(\frac{\lambda_0 + \mu_0 + 1}{\lambda + \mu + 1}\right)^2, \quad \text{for intraband transitions}; \tag{2}$$

$$e^2_{\text{eff-inter}} = \left(\frac{\mu_0 + 1}{\mu + 1}\right)^2, \quad \text{for interband transitions}. \tag{3}$$

The two dependencies, Eqs. (2) and (3), approximately show how the numerical scale of the original (unscaled) VBM intra- and interband B(E2) transition values departs from that in the favored SU(3)-irrep (λ_0, μ_0).

Now we repeated the VBM calculations with the SU(3)-irreps given in Table 1 by scaling the theoretical B(E2) values according to Eqs. (2) and (3) with the favored SU(3)-irrep (λ_0, μ_0)=(16,2). The result is shown in Table 2. We

Table 2. Energy and B(E2) RMS deviations σ_E (in keV) and σ_B (in W.u.) obtained for various (λ, μ)-irreps in ^{166}Er by using the scaling factors (2) and (3) with (λ_0, μ_0)=(16,2)

λ	μ	σ_E	σ_B	λ	μ	σ_E	σ_B	λ	μ	σ_E	σ_B
16	2	57.674	38.782	30	2	60.961	48.820	52	2	65.753	59.249
16	4	58.118	37.261	30	4	62.965	39.902	52	4	66.467	51.927
16	6	70.178	40.899	30	6	66.252	34.888	52	6	67.524	45.993
16	8	108.108	46.092	30	8	71.280	33.383	52	8	69.080	41.417
16	10	192.247	51.738	30	10	78.487	34.484	52	10	70.905	38.165
16	12	168.264	57.356	30	12	88.227	37.125	52	12	73.370	36.167
16	14	185.742	62.792	30	14	100.609	40.494	52	14	76.444	35.292

see that now all σ_B deviations enter the scale of the RMS factor in the favored SU(3)-irrep. The detailed inspection of the different described transitions shows that for all considered (λ, μ) irreps the $B^{\text{th}}(E2)/B^{\text{exp}}(E2)$ ratio only slightly varies around the unit value. In Table 2 we also see that the largest σ_E-values of the energy RMS deviations in the large-μ irreps at λ=16 and 30 are now obtained essentially smaller. This result suggests that the change in the scale of the B(E2)-values gives some improvement of the energy description in this region of SU(3)-irrep. However, as noticed above, these σ_E-values remain much larger compared to those in the other (λ, μ)-regions showing that the scaling can not (and should not) lead to description of a strong triaxial structure of the spectrum where it is not supposed to manifest.

The more detailed examination of the $B^{\text{th}}(E2)/B^{\text{exp}}(E2)$ ratios shows that the (λ, μ)-dependence in Eq. (2) may be also taken as

$$e^2_{\text{eff-intra}} = \left(\frac{\lambda_0 + \mu_0}{\lambda + \mu}\right)^2 . \tag{4}$$

The calculations made for the different (λ, μ) irreps with this scaling dependence also give a slight variation of the $B^{\text{th}}(E2)/B^{\text{exp}}(E2)$ ratio around the unit value. The only difference with the result obtained by using (2) is that (4) gives smaller/bigger deviations above/below the unit $B^{\text{th}}(E2)/B^{\text{exp}}(E2)$ value in the intraband transitions for the lower/upper parts of the ground- and γ-band level sequences along the series of considered SU(3) irreps. Given the practically equivalent effect of both approximate dependencies we retain for further tests in the present work the definition in Eq. (2).

It should be noted that the effective charges (2)–(4) do not introduce new parameters. They only depend on the already available SU(3)-irrep numbers λ and μ whose "favored" values λ_0 and μ_0 are determined by the structure of the spectrum as done in the original VBM algorithm [2]. An open question remaining at this point is about the possible physical origin/meaning of dependencies (2)–(4). They obviously compensate for the strong increase of the VBM B(E2)

transition strengths with the deviation of the SU(3)-irrep from the favored one. Deeper study in this direction remains to be done.

3 Numerical Algorithm with SU(3)-irrep-Dependent Effective Charges. Test on ^{166}Er.

Based on the results of Section 2, the numerical algorithm for the proposed approach of SU(3)-irrep-dependent effective charges includes the following two steps:

(i) Determination of the favored SU(3)-irrep (λ_0, μ_0) for which the VBM description gives minimal RMS deviations between the theoretical energy levels and B(E2) transition values and the corresponding experimental data without use of scaling factors in the B(E2)-values.

(ii) Inclusion of the (λ_0, μ_0)-values into the effective charge expressions Eqs. (2) [or (4)] and (3) and (re-)calculation of the energy spectrum and transition probabilities for the irrep(s) of interest with the so-determined scaling factors. At this step new values of the RMS deviations are obtained for each SU(3)-irrep and eventually new EMF irrep providing the best model description can be found.

A key question in the determination of the favored (λ_0, μ_0)-irrep in step (i) and the EMF irrep in step (ii) above is the criterion for the best model description. It refers to the simultaneous assessment of the energy and transition RMS factors σ_E and σ_B for which there is no unambiguous prescription at all. It may depend on the way in which these quantities are obtained in the fit. In our numerical fits we minimize the sum $\sigma_E + \sigma_B^w$, where in σ_B^w the differences $B(E2)_\nu^{\rm th} - B(E2)_\nu^{\rm exp}$ are multiplied by weighting factors used to provide approximately the same numerical impact of both transition rates and energy levels in

Table 3. Energy and B(E2) RMS deviations σ_E (in keV) and σ_B (in W.u.) and their sum σ_T (in relative units, r.u.) obtained for various (λ, μ)-values in ^{166}Er without use of B(E2) scaling factors. The lowest σ_E and σ_B values are underlined and that for σ_T is bolded

λ	μ	σ_E	σ_B	$\sigma_T = \sigma_E + \sigma_B$
16	2	57.674	<u>38.782</u>	**96.456**
18	2	<u>55.288</u>	75.648	130.936
20	2	55.580	141.684	197.264
22	2	56.651	217.700	274.351
16	4	76.169	50.112	126.281
18	4	73.889	108.928	182.817
20	4	58.454	108.052	166.506
16	6	70.519	81.042	151.561
18	6	129.101	148.538	277.639
20	6	65.608	224.970	290.578

the minimization procedure. Depending on the choice of these factors one may achieve better description of the B(E2)s on the expense of energy levels or vice versa. Therefore, this is an obvious source of unambiguity in the assessing of description quality. However, in our case it appears that in the (λ, μ)-regions where the favored irreps are expected to appear the non-weighted σ_B values in W.u. and σ_E in keV are obtained of the same order. On this basis we take as a criterion for the overall quality of the model description their direct sum $\sigma_T = \sigma_E + \sigma_B$ (total RMS factor) determined in relative units (r.u.).

Below we demonstrate the application of the above two-step analysis to the ground- and γ-band spectra and B(E2)-values in ^{166}Er [3]. At the first step we made series of VBM calculations without effective charges for various combinations of λ and μ extending the results in Table 1. The obtained σ_E and σ_B deviations and their sums σ_T are collected in Table 3. It is seen that among the considered (λ, μ)-irreps the minimal total RMS deviation $\sigma_T = 96.5$ r.u. is obtained for $(\lambda_0, \mu_0) = (16, 2)$ which appears to be (remain) the same favored SU(3)-irrep determined in Ref. [2] and used in the calculations for Table 2. It

Table 4. Energy and B(E2) RMS deviations σ_E (in keV) and σ_B (in W.u.) and their sum σ_T (in relative units, r.u.) obtained for various (λ, μ)-irreps in ^{166}Er by using the effective charges of Eqs. (2) and (3) with $(\lambda_0, \mu_0) = (16, 2)$ in the calculation of B(E2)-values. The squared effective charge values are also given

λ	μ	σ_E	σ_B	$e^2_{\text{eff-intra}}$	$e^2_{\text{eff-inter}}$	σ_T
16	2	57.674	38.782	1.000	1.000	96.456
18	2	55.286	37.427	0.819	1.000	92.713
20	2	55.580	38.527	0.682	1.000	94.107
22	2	56.548	40.571	0.578	1.000	97.119
24	2	57.866	42.836	0.495	1.000	100.702
26	2	59.021	45.024	0.429	1.000	104.045
28	2	60.063	47.027	0.376	1.000	107.090
16	4	58.118	37.261	0.819	0.360	95.379
18	4	57.861	33.830	0.682	0.360	91.691
20	4	58.645	32.945	0.578	0.360	**91.590**
22	4	59.658	33.537	0.495	0.360	93.195
24	4	60.643	34.870	0.429	0.360	95.513
26	4	61.527	36.509	0.376	0.360	98.036
28	4	62.299	38.224	0.331	0.360	100.523
16	6	70.178	40.899	0.682	0.184	111.077
18	6	67.127	36.421	0.578	0.184	103.548
20	6	66.192	33.845	0.495	0.184	100.037
22	6	65.890	32.694	0.429	0.184	98.584
24	6	65.862	32.529	0.376	0.184	98.391
26	6	65.953	32.995	0.331	0.184	98.948
28	6	66.092	33.839	0.295	0.184	99.931
52	14	76.444	35.292	0.080	0.040	111.736

should be noted that while this irrep gives slightly higher energy deviation of $\sigma_E = 57.7$ keV than the (18,2), (20,2) and (22,2) irreps it provides much better (lower) value of $\sigma_B = 38.8$ W.u. and subsequently the best σ_T-factor.

At the second step, calculations with the effective charges determined by the favored irrep (16, 2) are made. The results, extending the content of Table 2, are listed in Table 4. Here the lowest value of the total RMS factor $\sigma_T = 91.6$ r.u. (bolded in the table) is obtained at the irrep (20,4) which appears to be the EMF SU(3)-irrep for ^{166}Er. However, we notice that the value of $\sigma_T = 91.7$ r.u., obtained at the neighboring in λ irrep (18,4), is practically the same.

Therefore, it is instructive to examine the overall irrep dependence of the obtained model descriptions quality. The λ-dependence of the σ_E-values obtained for $\mu = 2$, 4 and 6 is shown in Figure 1(a). Here the $\mu = 2$ curve shows a minimum at $(\lambda, \mu) = (18, 2)$ followed by the slightly above placed $\mu = 4$ curve with a shallow minimum at (18,4). The result for σ_B is shown in Figure 1(b). Here a shallow minimum appears at (24,6) with very close values obtained at (22,6), (26,6) and (20,4). Subsequently, the total RMS factor σ_T also shows a smooth behaviour illustrated in Figure 1(c). Here the lowest σ_T-value at (20,4) together with the very close value at (18,4) form somehow flat minimum of the $\mu = 4$ curve. Closely above it is the $\mu = 2$ curve with a σ_T-minimum at (18,2). In this sense our calculations outline a group of several close to one another effectively favorable SU(3)-irreps for ^{166}Er corresponding to Figure 3 in Ref. [2].

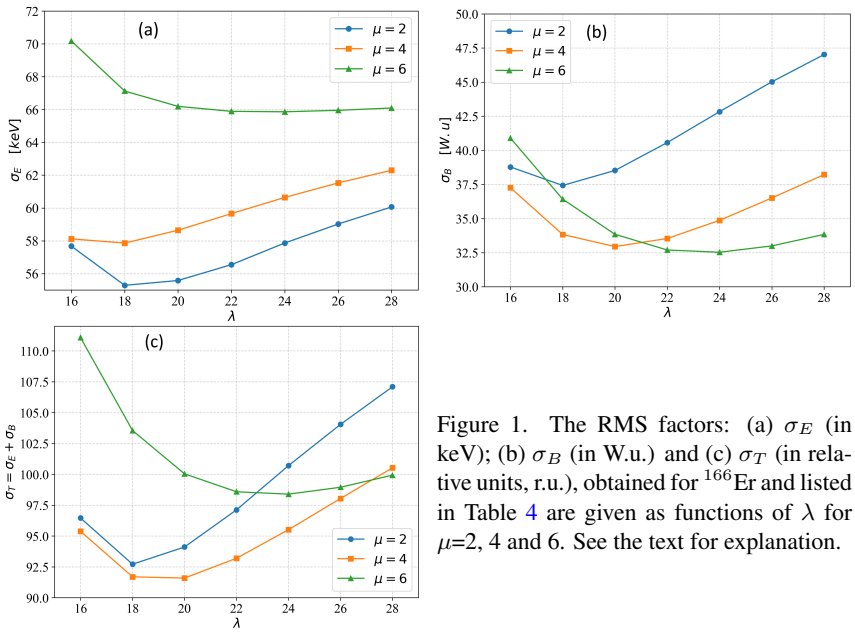

Figure 1. The RMS factors: (a) σ_E (in keV); (b) σ_B (in W.u.) and (c) σ_T (in relative units, r.u.), obtained for ^{166}Er and listed in Table 4 are given as functions of λ for μ=2, 4 and 6. See the text for explanation.

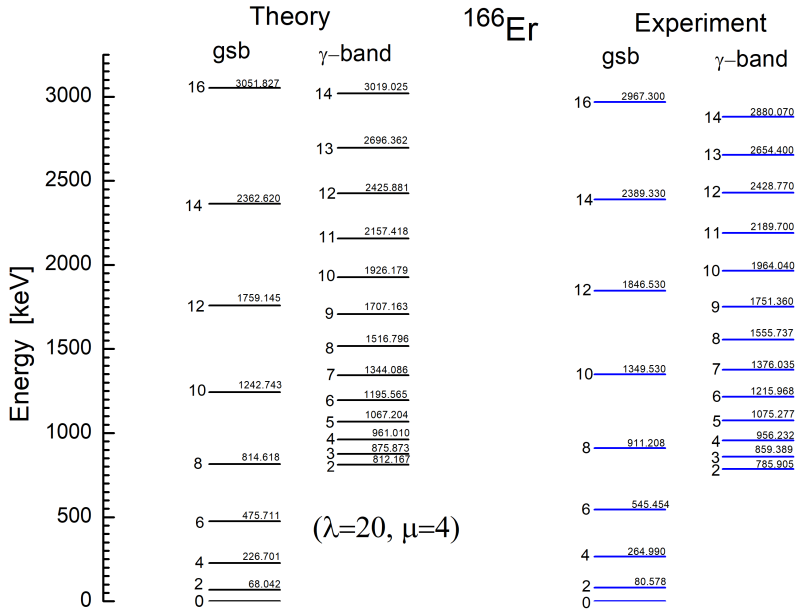

Figure 2. Theoretical and experimental energy levels of the ground- and γ- bands of ^{166}Er. The data are taken from [3]. The theoretical values are calculated for the EMF SU(3)-irrep (20,4) together with the B(E2) transition values obtained through the corresponding irrep-dependent effective charges (see Table 5).

On the bottom of Table 4 we give for comparison the result obtained for the proxy SU(3) irrep (52,14), already available in Table 1. It is seen that the model description in this irrep, with $\sigma_T = 111.7$ r.u., follows the overall quality of the VBM calculations in the considered range of SU(3)-irreps.

A comparison of the ground- and γ-band energy levels obtained for ^{166}Er in the EMF irrep (20,4) is given in Figure 2. We see that the model quite well reproduces the overall structure of the spectrum including the high angular momenta. The comparison between the B(E2) transition values obtained in the same VBM calculation and the experimental data is presented in Table 5. Again, we see quite good overall reproduction of the considered experimental data on the gsb and γ-band intra- and interband B(E2)-values.

Finally, a comment is due to an obvious effect of the introduced irrep-dependent charges on the result of VBM calculation in ^{166}Er. This is the shift from the more symmetric favoured SU(3)-irrep (16,2) of the unscaled calculation to the less symmetric EMF irrep (20,4) obtained in the "scaled" solution. From shape deformation point of view the new solution corresponds to the recognition of some (not large) amount of triaxiality in the structure of the ^{166}Er spectrum. It is interesting that this result somehow points towards the stronger triaxiality

Table 5. Theoretical and experimental B(E2) transition probabilities (in W.u.) and their ratios for ^{166}Er. The theoretical values are calculated with the effective charges determined for the EMF irrep $(20,4)$. The data are taken from [3]. The uncertainties (in brackets) refer to the last digits of the experimental values

L_{b_i}	L_{b_f}	$B(E2)_{\text{th}}$	$B(E2)_{\text{exp}}$	$B(E2)_{\text{th}}/B(E2)_{\text{exp}}$
2g	0g	254.613	217(5)	1.17
4g	2g	358.137	312(11)	1.15
6g	4g	383.41	370(20)	1.04
8g	6g	384.777	373(14)	1.03
10g	8g	373.154	390(17)	0.96
12g	10g	352.329	372(21)	0.95
2γ	0g	4.892	5.17(21)	0.95
2γ	2g	8.299	9.6(6)	0.86
2γ	4g	0.608	0.78(4)	0.78
4γ	2g	2.285	1.98(12)	1.15
4γ	6g	1.593	2.1(14)	0.76
6γ	4g	1.438	0.88(6)	1.63
6γ	8g	2.486	1.9(3)	1.31
8γ	6g	0.954	0.52(5)	1.83
8γ	10g	3.412	1.5(0)	2.27
4γ	2γ	148.027	138(9)	1.07
4γ	3γ	335.804	370(3)	0.91
5γ	3γ	234.85	300(4)	0.78
5γ	4γ	242.793	310(4)	0.78
6γ	4γ	280.643	225(16)	1.25
8γ	6γ	318.545	250(23)	1.27
9γ	7γ	331.105	370(15)	0.89

predicted by the proxy-SU(3) irrep (52,14). Further more detailed study involving quantitative estimations of the related γ-deformation [6] and its connection to the obtained (λ,μ)-dependencies of the B(E2) transition probabilities could shed light on some deeper effects of the SU(3) symmetry in the collective and intrinsic structure of heavy deformed nuclei.

4 Conclusion

In this work a modified numerical implementation of the VBM with broken SU(3) symmetry is presented. The model description of the ground- and γ-intra- and interband B(E2) transition probabilities, which so far was obtained in the form of B(E2)-ratios, is now provided by direct comparison of theoretical and experimental B(E2)-values. This is achieved through introduction of effective charges depending on the SU(3)-irrep numbers λ and μ without introduction of new model parameters. The numerical calculations performed for the ground- and γ-band spectra in ^{166}Er demonstrate the relevance of the introduced

scaling factors for the realistic theoretical description of the B(E2) transition values within VBM. The relevance of the VBM calculations using the physically justified proxy-SU(3) irrep was pointed out. Further work has to be done to validate the use of SU(3)-irrep dependent effective charges in other nuclei, as well as, to clarify their physical meaning.

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