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Double-Beta Decay of ^{48}Ca within the STDA Method Based on Realistic Interaction with Density-Dependent Corrective Term

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Abstract. We investigate nuclear matrix elements for two-neutrino double-beta decay of ^{48}Ca using the Second Tamm-Dancoff method. This approach employs a realistic chiral nucleon-nucleon interaction, known as NNLO$_{\text{opt}}$, which is supplemented by phenomenological density-dependent terms that incorporate both isoscalar ($T = 0$) and isovector ($T = 1$) channels. We adjust the coupling constants for these channels to minimize the corresponding Fermi matrix element. This value is negligible compared to the Gamow-Teller nuclear matrix element, suggesting the restoration of isospin symmetry. Our calculations provide a foundation for a more detailed study, in which we will address the issue of quenching the weak coupling constant g_A.

1 Introduction

Double-beta decay is a process that lies at the intersection of particle physics, nuclear physics, and atomic physics, offering valuable insights into concepts beyond the Standard Model. There are two types of double-beta decay processes: two-neutrino double-beta ($2\nu\beta\beta$) decay and neutrinoless double-beta ($0\nu\beta\beta$) decay. The $0\nu\beta\beta$ decay occurs only if neutrinos are Majorana particles, whereas the $2\nu\beta\beta$ decay can happen regardless of whether the neutrinos are Dirac or Majorana particles. Observing $0\nu\beta\beta$ decay would uncover new fundamental properties of neutrinos. If $0\nu\beta\beta$ decay is detected, it would prove that lepton number conservation is not a fundamental conservation law, which has profound implications for our understanding of the observed matter-antimatter asymmetry in the Universe.

A theoretical description of $0\nu\beta\beta$ decay requires a precise understanding of nuclear structure, which is essential for evaluating related nuclear matrix elements (NMEs) and their associated uncertainties. Current methods for calculating NMEs, such as the Shell Model, Interacting Boson Model, and Quasiparticle Random Phase Approximation (QRPA), often yield very different results, with discrepancies as large as a factor of 2 to 4 for a given $\beta\beta$ transition [1]. Variations in the effective axial-vector coupling constant, g_A, further complicate a reliable description of double beta decay. Studying the $2\nu\beta\beta$ decay, along with other nuclear processes such as ordinary muon capture, nucleon transfer reactions, double-gamma decay, single-charge exchange, and double-charge exchange reactions, using nuclear structure methods that involve more complex configurations can help reduce uncertainties associated with calculating NMEs. While these studies do not directly access the NMEs for $0\nu\beta\beta$ decay, they provide valuable information that contributes to this objective [1].

In this contribution, we utilize the Second TammDancoff Approximation, which has been successfully employed to describe electromagnetic nuclear excitations [2], to calculate double-beta-decay transitions. We apply this method to evaluate the matrix elements governing the $2\nu\beta\beta$ decay of ^{48}Ca. Our study relies on the realistic chiral potential NNLO$_{\rm opt}$, supplemented by a density-dependent interaction term that includes both isoscalar and isovector components.

2 Theoretical Formalism

We employ three nuclear structure methods in this study: Hartree-Fock (HF), Tamm-Dancoff Approximation (TDA), and Second Tamm-Dancoff Approximation (STDA).

By solving the HF method [3], we obtain the self-consistent single-particle (s.p.) basis and nuclear Hamiltonian in the form

$$\hat{H} = E_{\rm HF} + \sum_i \varepsilon_i : a_i^\dagger a_i : + \frac{1}{4} \sum_{ijkl} V_{ijkl} : a_i^\dagger a_j^\dagger a_l a_k :, \tag{1}$$

where $E_{\rm HF}$, ε_i and V_{ijkl} stand for Hartree-Fock energy of the studied nucleus, single-particle energies, and the interaction matrix elements, respectively. a_i ($a_i^\dagger$) is a nucleon annihilation (creation) operator. The HF wave function $|\Psi_{\rm HF}\rangle$ of the studied nucleus is represented by a single Slater determinant composed of the lowest single-particle states occupied by protons and neutrons.

In the TDA method [3], we diagonalize the nuclear Hamiltonian (1) within the space spanned by all 1-particle–1-hole (1p-1h) configurations $a_p^\dagger a_h |\Psi_{\rm HF}\rangle$, where the index p (h) denotes all unoccupied (occupied) single-particle levels. The corresponding eigenvalue equation is given by:

$$\sum_{p'h'} \left((\varepsilon_p - \varepsilon_h) \delta_{pp'} \delta_{hh'} + V_{p'hh'p} \right) c^\mu_{p'h'} = (E^{\rm TDA}_\mu - E_{\rm HF}) c^\mu_{ph}, \tag{2}$$

where $E^{\rm TDA}_\mu$ represents the nuclear eigenenergies, while c^μ_{ph} denotes the corresponding TDA amplitudes. It is important to note that the eigenenergies on the right side of Eq. (2) are referenced relative to the HF energy, specifically as $E_{\mu}^{\mathrm{TDA}} - E_{\mathrm{HF}}$. In our formalism, 1p-1h excitations also include configurations that transform a neutron into a proton and *vice versa*. Consequently, we obtain not only the eigenenergies for the (A, Z) nucleus but also those associated with the $(A, Z+1)$ and $(A, Z-1)$ nuclei.

In the STDA method [4], we diagonalize the nuclear Hamiltonian (1) within the space spanned by all 1p-1h configurations, represented as $a_p^{\dagger} a_h |\Psi_{\mathrm{HF}}\rangle$, as well as 2p-2h configurations, denoted by $a_{p_1}^{\dagger} a_{p_2}^{\dagger} a_{h_1} a_{h_2} |\Psi_{\mathrm{HF}}\rangle$. By solving the corresponding eigenvalue equation, discussed in [4], we obtain the eigenstates,

$$|\mathrm{STDA}; \mu\rangle = \left(\sum_{ph} X_{ph}^{\mu} a_p^{\dagger} a_h + \sum_{p_1<p_2} \sum_{h_1<h_2} \mathcal{X}_{p_1 p_2 h_1 h_2}^{\mu} a_{p_1}^{\dagger} a_{p_2}^{\dagger} a_{h_1} a_{h_2} \right) |\Psi_{\mathrm{HF}}\rangle \tag{3}$$

and the eigenenergies E_{μ}^{STDA}. Analogous to the TDA method, the relative energies $(E_{\mu}^{\mathrm{STDA}} - E_{\mathrm{HF}})$ are introduced. The solutions obtained are linked not only to the (A, Z) nucleus but also to the $(A, Z \pm 1)$ and $(A, Z \pm 2)$ nuclei. This is because we also include particle-hole configurations that can convert neutrons to protons (or *vice versa*).

The $2\nu\beta\beta$ Fermi (F) and GamowTeller (GT) NMEs can be written as

$$M_{F/GT} = \sum_{\lambda} \frac{\langle f || \hat{O}_{F/GT} || \lambda \rangle \langle \lambda || \hat{O}_{F/GT} || \Psi_{\mathrm{HF}} \rangle}{(E_{\lambda}^{\mathrm{TDA}} - E_{\mathrm{HF}}) - \frac{1}{2}((E_{\lambda}^{\mathrm{STDA}} - E_{\mathrm{HF}}) - E_{\mathrm{HF}})} = \\ = \sum_{\lambda} \frac{\langle f || \hat{O}_{F/GT} || \lambda \rangle \langle \lambda || \hat{O}_{F/GT} || \Psi_{\mathrm{HF}} \rangle}{E_{\lambda}^{\mathrm{TDA}} - \frac{1}{2} E_{\lambda}^{\mathrm{STDA}}}, \tag{4}$$

where $|\Psi_{\mathrm{HF}}\rangle$ represents the HF Slater determinant of the parent nucleus, $|\lambda\rangle$ denotes the wave functions of the intermediate nucleus $(A, Z+1)$ obtained using the TDA method, and $|f\rangle$ is the ground state of the daughter nucleus $(A, Z+2)$ within the STDA framework. The Fermi operator, denoted as $\hat{O}_F = \tau^+$, and the GamowTeller operator, represented as $\hat{O}_{GT} = \tau^+ \sigma_1$, play crucial roles in $2\nu\beta\beta$ transitions. Here, τ^+ is responsible for converting a neutron into a proton, while σ_1 is the vector operator associated with the Pauli spinor. The ground state of an eveneven nucleus (A, Z) possesses zero spin and even parity. Thus, for both the Fermi and GamowTeller transitions, the states $|\lambda\rangle$ of the intermediate nucleus exhibit multipolarities of 0^+ (for Fermi transitions) and 1^+ (for GamowTeller transitions).

For the $2\nu\beta\beta$-decay half-life, we have

$$(T_{1/2}^{2\nu})^{-1} = g_A^4 m_e^2 \left| M_{GT} - \frac{M_F}{g_A^2} \right|^2 G^{2\nu}, \tag{5}$$

where $G^{2\nu}$ is the phase-space factor [1]. The weak coupling constant is denoted as $g_A = 1.27$ and m_e refers to the mass of the electron. Further, we introduce the total $2\nu\beta\beta$ matrix element:

$$M_{\rm tot} = M_{GT} - \frac{M_F}{g_A^2}. \quad (6)$$

3 Calculations

We implement the intrinsic nuclear Hamiltonian, $H = T + V$, obtained by subtracting the center-of-mass (CM) term. The corrected one-body kinetic part reads

$$T = \left(1 - \frac{1}{A}\right) \frac{1}{2m} \sum_i \vec{p}_i^{\,2}, \quad (7)$$

while the two-body kinetic term

$$T_2 = -\frac{1}{mA} \sum_{i<j} \vec{p}_i . \vec{p}_j \quad (8)$$

is incorporated into the nucleon-nucleon (NN) interaction V defined as

$$V = T_2 + V_{\rm opt}^{NN} + V_\rho^{NN}. \quad (9)$$

The term $V_{\rm opt}^{NN}$ refers to the nucleon-nucleon interaction component of the chiral potential $\rm NNLO_{opt}$ [5], which is a realistic interaction optimized to minimize the effects of the three-body NNN force.

V_ρ^{NN} represents a phenomenological density-dependent (DD) nucleon-nucleon interaction [6], which consists of both isoscalar ($T = 0$) and isovector ($T = 1$) components as follows:

$$V_\rho^{NN} = \frac{C_\rho^{T=0}}{6}(1 + \hat{P}_\sigma)\rho\left(\frac{\vec{r}_1 + \vec{r}_2}{2}\right)\delta(\vec{r}_1 - \vec{r}_2) + \frac{C_\rho^{T=1}}{6}(1 - \hat{P}_\sigma)\rho\left(\frac{\vec{r}_1 + \vec{r}_2}{2}\right)\delta(\vec{r}_1 - \vec{r}_2), \quad (10)$$

where $\hat{P}_\sigma$ is the operator representing spin exchange, and $C_\rho^{T=0}$ (for the isoscalar part) and $C_\rho^{T=1}$ (for the isovector part) denote their respective coupling constants. Note that the $T = 0$ and $T = 1$ terms are associated with the total spin S = 1 and S = 0, respectively, due to the antisymmetrization of the entire DD interaction.

The calculations for the HF, TDA, and STDA methods were conducted using the harmonic oscillator basis, which spans major oscillator shells ranging from $N = 0$ to $N = 6$, encompassing a total of 28 j-levels. The basis parameter was selected as $\hbar\omega = 10$ MeV.

4 Results

We study the $2\nu\beta\beta$ of ^{48}Ca, i.e., the nuclear process

$$^{48}\text{Ca} \rightarrow^{48} \text{Ti} + 2\text{e}^- + 2\bar{\nu}_\text{e}. \tag{11}$$

This second-order transition occurs through the virtual states of the intermediate nucleus ^{48}Sc.

We exploit the HF method to obtain the single-particle energies ε_i and the Slater determinant $|\Psi_\text{HF}\rangle$ for ^{48}Ca. Further, we proceed with calculating the energy spectra of ^{48}Sc and ^{48}Ti using the TDA and the STDA methods, respectively. To achieve this, we systematically adjust the proton energies ε_i to ensure that $\varepsilon^\text{neut}_{0f_{7/2}} - \varepsilon^\text{prot}_{0f_{7/2}} \approx 1.8$ MeV. Finally, we compute the Fermi and Gamow-Teller NMEs using Eq. (4), adjusting the TDA and STDA energies to match the experimental ground-state energies of ^{48}Sc and ^{48}Ti.

At the beginning of the calculation, we turn off the DD term in the interaction. By applying the TDA method to ^{48}Sc we obtain the energy difference $(E^\text{TDA}_{6^+_1} - E_\text{HF}) = -1.475$ MeV, which is by 0.6845 MeV lower as it follows the measured value $Q_\beta = 0.2795$ MeV [7]. The STDA method, used for ^{48}Ti, gives $(E^\text{STDA}_{0^+_1} - E_\text{HF}) = -3.627$ MeV, i.e. 1.663 MeV higher value as one get by using experimental value $Q_{\beta\beta} = 4.268$ MeV [7]. We end up with $M_F = 3.13 \times 10^{-2}$ MeV^{-1}, and $M_{GT} = 1.14 \times 10^{-2}$ MeV^{-1}, which indicate a significant breaking of the isospin symmetry, as these quantities are comparable. Our main interest lies in whether isospin symmetry can be restored by adjusting the DD part of the interaction V^{NN}_ρ. As a matter of fact, we found a significant dependence of the Fermi and Gamow-Teller NMEs on both coupling constants $C^{T=0}_\rho$ and $C^{T=1}_\rho$.

We proceed by setting $C^{T=0}_\rho = -900.0$ MeV.fm^6, and $C^{T=1}_\rho = -600.0$ MeV.fm^6. Consequently, for ^{46}Sc we calculate that $(E^\text{TDA}_{6^+_1} - E_\text{HF}) = -2.190$ MeV, which is by 1.400 MeV lower than the value derived from Q_β [7]. For ^{48}Ti, we find $(E^\text{STDA}_{0^+_1} - E_\text{HF}) = -4.122$ MeV, which is 1.168 MeV higher than determined by measured by $Q_{\beta\beta}$ [7]. The calculation yield $M_F = 8.54 \times 10^{-3}$ MeV^{-1} and $M_{GT} = 0.172$ MeV^{-1}. We conclude that isospin symmetry is nearly restored. For the total matrix element, we obtain $M_\text{tot} = 0.180$ MeV^{-1}, which implies $2\nu\beta\beta$ decay half-life $T^{2\nu}_{1/2}(^{48}Ca) = 2.922 \times 10^{18}$ years. To match the experimental half-life value of $T_{1/2} = 5.3 \times 10^{19}$ years [8], we must apply a quenching factor of $q = 0.51$ to the weak coupling constant $g_A = 1.27$. These results are consistent with our previous calculations of $2\nu\beta\beta$ of ^{48}Ca using the STDA method [9, 10].

We demonstrate that by properly adjusting the coupling constants $C^{T=0}_\rho$ and $C^{T=1}_\rho$, we can minimize the Fermi NME M_F, effectively restoring isospin symmetry. This is crucial for our next objective: incorporating 2-particle–2-hole STDA configurations for not only the daughter nucleus ^{48}Ti, but also for the parent nucleus ^{48}Ca and the intermediate nucleus ^{48}Sc. We anticipate that this approach will significantly account for the quenching effect of the g_A coupling constant. Additionally, we plan to address the issue of center-of-mass spuriousness in the STDA energy spectrum, which we will tackle using the SVD method [11].

5 Conclusions

In summary, we investigated the $2\nu\beta\beta$ decay of ^{48}Ca using the HF, TDA and STDA methods. We employed a realistic chiral nucleon-nucleon interaction, $\mathrm{NNLO_{opt}}$, along with a density-dependent interaction term that includes both isoscalar and isovector channels. Our results showed that by adjusting the strength of these channels, we could minimize the Fermi NME relative to the Gamow-Teller NME, indicating a restoration of isospin symmetry.

Our future goals include incorporating 2-particle–2-hole configurations into the description of the parent nucleus ^{48}Ca and the intermediate nucleus ^{48}Sc. Additionally, we aim to address the issue of center-of-mass spuriousness in the STDA energy spectrum by applying the SVD method.

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References

[1] J. Engel, J. Menendez, *Rep. Prog. Phys.* **80** (2017) 046301.

[2] F. Knapp, P. Papakonstantinou, P. Veselý , G. De Gregorio, J. Herko, N. Lo Iudice, *Phys. Rev. C* textbf107 (2023) 014305.

[3] P. Ring, P. Schuck, *Nuclear Many Body Problem* (Springer-Verlag, New York 1980); Chaps. 5, 8.

[4] P. Papakonstantinou, R. Roth, *Phys. Rev. C* **81** (2010) 024317.

[5] A. Ekström, G. Baardsen, C. Forssén, G. Hagen, M. Hjorth-Jensen, G.R. Jansen, R. Machleidt, W. Nazarewicz, T. Papenbrock, J. Sarich et al., *Phys. Rev. Lett.* **110** (2013) 192502.

[6] D. Bianco, F. Knapp, N. Lo Iudice, P. Veselý, F. Andreozzi, G. De Gregorio, A. Porrino, *J. Phys. G: Nucl. Part. Phys.* **41** (2014) 025109.

[7] NuDat3.0 database, https://www.nndc.bnl.gov/nudat3/.

[8] A. Barabash, *Universe* **6** (2020) 159.

[9] P. Veselý, D. Denisova, F. Knapp, F. Šimkovic, *Acta Phys. Pol. B Proc. Suppl.* **18** (2025) 2-A9.

[10] P. Veselý, G. De Gregorio, D. Denisova, F. Knapp, N. Lo Iudice, F. Šimkovic, "Double Beta Decay of ^{48}Ca within EMPM and STDA methods", *Proceedings of the conference "14th International Spring Seminar on Nuclear Physics: Cutting-edge developments in nuclear structure physics"*, (to be published).

[11] G. De Gregorio, F. Knapp, N. Lo Iudice, P. Veselý, *Phys. Lett. B* **821** (2021) 136636.