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Manifestations of Triaxiality in Mo and Ru Nuclei

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Abstract. We investigate the presence of triaxial deformation in Mo and Ru isotopes. We present experimental indicators of triaxiality and address the issue of the distinction between soft and rigid triaxial behavior. Quantitative results for the nuclei under study are obtained by means of the Algebraic Collective Model, as well as microscopic Hartree-Fock-Bogoliubov calculations with Skyrme energy density functionals.

1 Introduction

The geometric collective model of Bohr and Mottelson [1, 2] predicts axially asymmetric deformations for nuclei when the shape variable γ takes values between the two axially symmetric cases $\gamma = 0^\circ$ (prolate shapes) and $\gamma = 60^\circ$ (oblate shapes), while $\beta \neq 0$. Not long after its introduction by A. Bohr in 1952, two models describing two extreme cases of triaxiality in even-even nuclei were proposed. The first one was the γ-unstable model by Wilets and Jean [3] where the deformation potential is independent of γ and the second was the γ-rigid rotor model by Davydov and Filippov [4], where γ has a fixed value. In what follows we examine the spectroscopic features of these two limiting cases and focus on Mo and Ru isotopes for which we will also provide calculations using both microscopic and macroscopic frameworks. A recent review on nuclear triaxiality can be found in [5]. The results presented here are part of a more extensive study [6].

2 Spectroscopic Features

The aforementioned models carry some very distinct spectral features. In the Wilets and Jean model the absence of γ from the potential leads to characteristic degeneracies that group the levels as $(3^+, 4^+), (5^+, 6^+), \ldots$ [7]. In the rigid-rotor model of Davydov and Filippov a quasi-γ band comes down in energy as γ increases from $0°$ to $30°$, *i.e.* from zero to maximum triaxiality, forming approximate doublets $(2^+, 3^+), (4^+, 5^+), \ldots$ [7]. Thus, by examining the level spacings in the γ band we can extract information about the degree of γ-softness / rigidity of the nucleus. For example, the *staggering* which is defined as

$$S(L) = \frac{E(L_\gamma^+) + E[(L-2)_\gamma^+] - 2E[((L-1)_\gamma^+]}{E(2_1^+)} \tag{1}$$

results in a zigzag pattern with minima at even values of L in the γ-unstable case and with minima at odd L values in the γ-rigid case.

It is worth noting that in the Davydov model the value of γ can be obtained directly from spectroscopic quantities [7]. More specifically,

$$\gamma = \frac{1}{3}\sin^{-1}\left(\frac{3}{R+1}\sqrt{\frac{R}{2}}\right), \tag{2}$$

where R is the energy ratio

$$R = \frac{E(2_2^+)}{E(2_1^+)}\,. \tag{3}$$

Another relation uses the B(E2) branching ratio R_2

$$R_2 = \frac{B(E2; 2_2^+ \to 2_1^+)}{B(E2; 2_2^+ \to 0_1^+)} = \frac{20}{7}\frac{\dfrac{\sin^2 3\gamma}{9 - 8\sin^2 3\gamma}}{1 - \dfrac{3 - 2\sin^2 3\gamma}{\sqrt{9 - 8\sin^2 3\gamma}}}\,. \tag{4}$$

In the next section we will see how the odd-even staggering in the γ band evolves over a series of Mo and Ru isotopes.

3 Algebraic Collective Model Calculations

The algebraic collective model (ACM) [8, 9] is a computationally tractable version of the collective model of Bohr and Mottelson. Matrix elements can be analytically calculated by exploiting the model's algebraic structure, which is that of the $\mathrm{SU}(1,1) \times \mathrm{SO}(5)$ dynamical group. An appropriate basis of wave functions is used to diagonalize the following Hamiltonian (see also [10])

$$\hat{H}(B, \alpha, \kappa, \chi; \beta, \gamma) = \frac{-\nabla^2}{2B} + \frac{1}{2}B[(1-2\alpha)\beta^2 + \alpha\beta^4] - \chi\beta^3\cos 3\gamma + \kappa\cos^2 3\gamma\,, \tag{5}$$

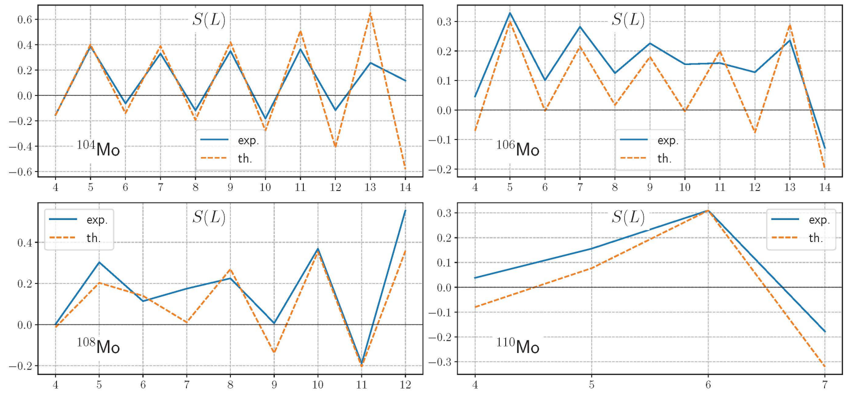

Figure 1. Staggering pattern in the γ bands of Mo isotopes, obtained from ACM calculations and compared with experiment. Adapted from [6].

where

$$\nabla^2 = \frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2}\hat{\Lambda} \tag{6}$$

is the Laplacian on the 5-dimensional collective model space and $\hat{\Lambda}$ is the SO(5) angular momentum operator. The B, α, κ and χ parameters are fitted to experimental data. The resulting spectra are in good agreement with experiment and as can be seen from the staggering patterns (Figures 1 and 2) a change in the type of triaxiality seems to occur at $N \approx 66$ neutron number, both for Mo and Ru isotopes. In particular, the minima of the odd-even staggering in the γ band, shift form even values of L (γ-soft) to odd (γ-rigid). This tendency becomes clear for higher values of angular momentum ($L > 8$).

4 Microscopic Self-Consistent Calculations

Deformation energy surfaces for the nuclei in question were obtained by means of microscopic calculations with Skyrme energy density functionals. More specifically, the MOCCa code [11–15] was used to solve the Hartree-Fock-Bogoliubov equations in a 3d cartesian coordinate-space representation and adjust the parameters of the various Skyrme interactions. Out of a number of Skyrme parameterizations, the BSkGx family (see [16–18] for examples) are found to give results that are closest to the available empirical data for Mo and Ru isotopes.

As can be seen in Figures 3 and 4, in both cases, the minima are more localized in the heavier isotopes ($A \gtrsim 114$), implying rigidity, while in the lighter ones the minima occupy a wider region implying softness. Another interesting feature is that for the $^{102-108}$Mo isotopes the minimum lies at γ slightly below

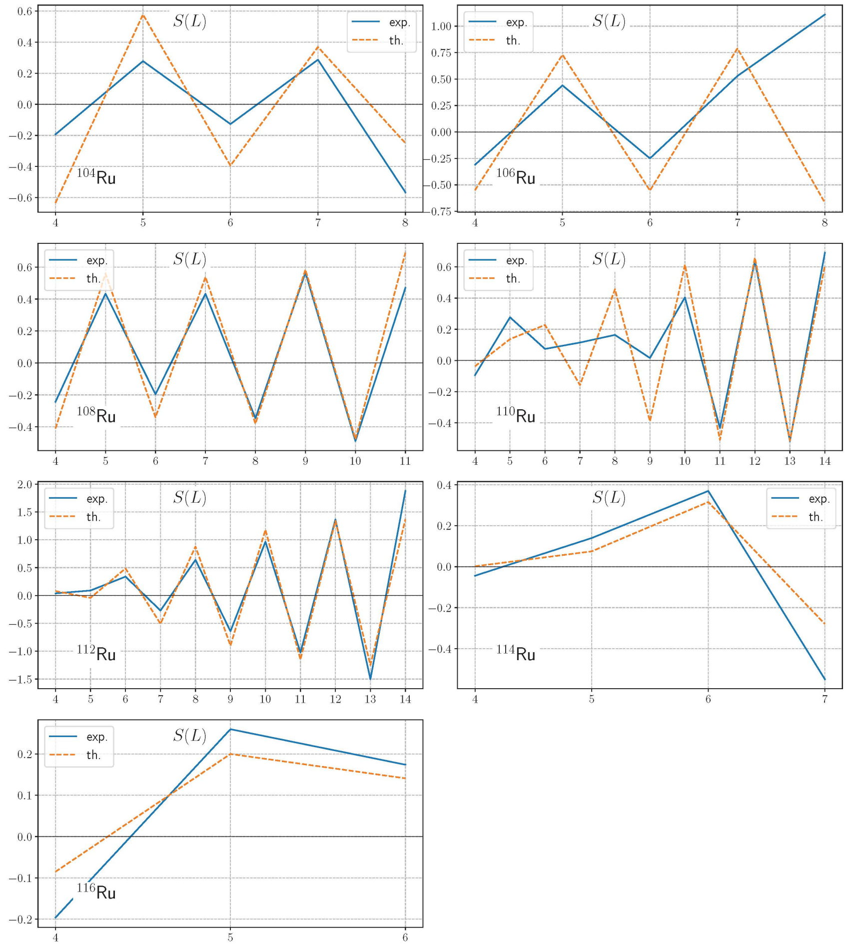

Figure 2. Staggering pattern in the γ bands of Ru isotopes, obtained from ACM calculations and compared with experiment. Adapted from [6].

$30°$, while for $^{110-116}$Mo it shifts to γ values over $40°$. Exactly the same behavior is observed between the two groups comprising $^{104-108}$Ru and $^{110-118}$Ru.

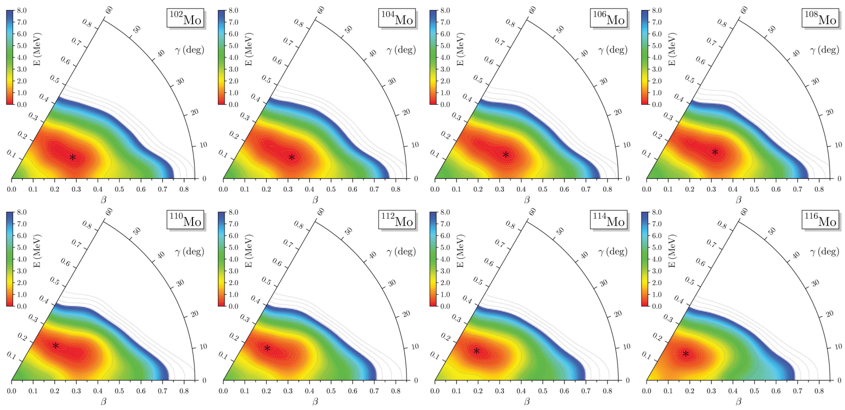

Figure 3. Deformation energy surfaces for $^{102-116}$Mo isotopes, resulting from Skyrme-HFB calculations with the BSkG2 parametrization. Adapted from [6].

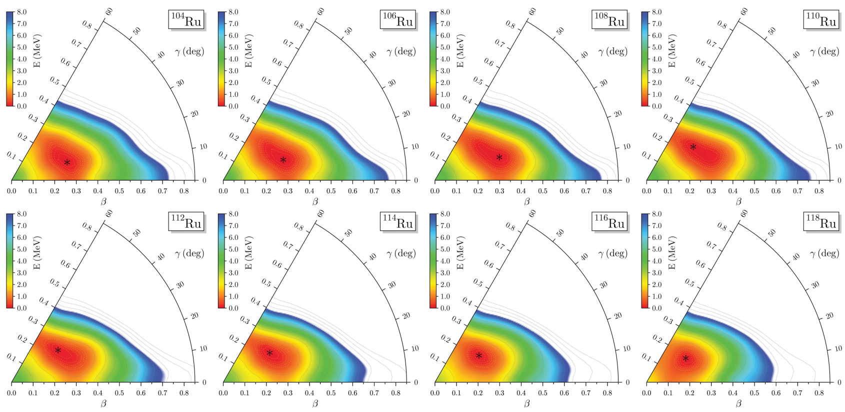

Figure 4. Deformation energy surfaces for $^{102-116}$Ru isotopes, resulting from Skyrme-HFB calculations with the BSkG2 parametrization. Adapted from [6].

5 Conclusions

In the present work we studied the evolution of triaxiality in Mo and Ru isotopes in the regions with neutron numbers above $N = 60$. We used spectroscopic signatures of triaxiality as a guide between the two extreme cases of rigid triaxial and γ-unstable rotor. Empirical observations, in particular the staggering in the γ band, indicate an onset of rigid triaxiality at $N \approx 66$ (midshell). Microscopic calculations with Skyrme energy density functionals (especially the BSkGx models) show triaxial minima with a sudden change in $\gamma_{\min}$ at $N \approx 66$ for Mo and Ru nuclei, while the area around the minimum becomes more confined at slightly higher masses, implying a trend toward rigidity. Concluding, we can say that the presence of triaxiality in the neutron-rich isotopes of Mo and Ru is established as well as a sudden change in its character around the neutron midshell ($N \approx 66$).

Acknowledgements

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